CFD Letters

Journal homepage:
https://semarakilmu.com.my/journals/index.php/CFD_Letters/index
ISSN: 2180-1363

Computational Simulation of Magneto Convection Flow of Williamson Hybrid Nanofluid with Thermal Radiation Effect

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ARTICLE INFO

Article history:
Received 6 October 2022
Received in revised form 8 November 2022
Accepted 9 December 2022
Available online 1 April 2023

Keywords:
Williamson hybrid nanofluid; MHD; stretching sheet;thermal radiation

ABSTRACT

The theme of this model is to examine the characteristics of heat and mass transfer flow through stretching sheet along with magnetic field and thermal radiation utilizing Al_2O_3+CuO/SA Williamson hybrid nanofluid. The transformed partial differential equations are solved by Keller-Box method. The numerical outcomes of physical quantities are revealed by graphs and tables. The Nusseltnumber, skin friction, velocity and temperature are displayed with support of bar diagram. The study depicted that an increase in the Weissenberg number, radiation, and magnetic parameter surges in fluid temperature, results in an improvement in the thermal boundary layer, this effect reduces the fluid velocity and skin friction coefficient. Excellent correctness of the current results has been acquired as compared thru the previous results.

1. Introduction

The Williamson fluid is a significant example of a non-Newtonian fluid; Williamson [1] presents model equations to describe the flow of pseudoplastic fluids and gives experimental proof of the conclusions in (1929). It has several potential uses, including in the photographic film emulsion coating and liquid film condensation processes. In addition, this non-Newtonian fluid has several industrial uses, particularly in the realm of pseudoplastic fluid behavior. Measuring mass and heat transport across the arteries in blood and hemodialysis is very crucial in the field of biological engineering [2]. Radiative Williamson fluid was investigated for its response to a magnetic field by Subbarayudu *et al.*, [3]. For their study, Sucharitha *et al.*, [4] examined how slip affected the peristaltic motion of a Williamson fluid. Dawar *et al.*, [5] provided an illustration of the flow of a Williamson nanofluid through a cone under non-isosolutal and non-isothermal circumstances. Two-dimensional Williamson nanofluid flow in stretchy sheets was investigated using Darcy's law by Kiyani *et al.*, [6]. Qureshi [7] looked at how thermal radiation affected Williamson nanofluid's heat transport. Recent studies related to Williamson nanofluid are presented in the Refs [8-11].

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https://doi.org/10.37934/cfdl.15.4.92105

The study of nanofluids is highly regarded in the scientific community. Nanomaterials with exceptional thermal characteristics may be used in a wide range of fields, from medicine and energy generation to heat exchange and electronic cooling systems. Due to its superior thermal conductivity, nanofluid has replaced base fluid as the preferred working fluid in many modern applications. Choi and Eastman [12] are credited with discovering nanofluids in the first place. Nanofluids are fluids that contain nanoparticles floating inside of them. Lee *et al.*, [13] verified that nanofluids exhibit superior heat transfer properties compared to those of basic liquids. Chemically stable oxides (SiO_2, CuO, Al_2O_3, TiO_2), carbides (SiC), metals (Au, Cu, Fe, Al, Ag), non-metals (carbon nanotubes, graphite), nitrides (AlN, SiN) are commonly proposed for the nanoparticles, and the base fluid is typically a conductive fluid such as water, oils (and further lubricants), propylene-gly Numerous researchers [14-17] have since shown that the nanofluids exhibit superior thermo-physical properties and heat-transport behavior compared to their base fluid counterparts. Adding copper nanoparticles to oil or ethylene glycol at a volume fraction of less than one percent, as determined by Eastman *et al.*, [14], increases thermal conductivity by 40 %. As Aybar*et et al.*, [15] noted, several nanofluids have improved their heat conductivity. Increasing the number of nanoparticles in a liquid, they found, increases its thermal conductivity. Nanofluids have several uses, including the administration of nano-drugs, the rapid transfer of heat, the cooling of a microchannel, and the prevention of clogging. Afify and Bazid [16] studied the effects of viscous dissipation and changing viscosity on the heat transfer and boundary layer flow on a rotating porous surface immersed in nanofluids. The effects of the particle model and thermal radiation on the heat transmission and Marangoni boundary sheet flow of a nanofluid moving through an exponential temperature were demonstrated by Lin *et al.*, [17].In recent times, A number of studies [18-23] examined the flow of a (MHD) Casson nanofluid by a stretching sheet, solid sphere and horizontal circular cylinder.

A hybrid nanofluid (HNF) is created by mixing two and more different nanoparticles in the same base fluid in order to create a (HNF). Most of the applications of this fluid are in the field of industry and manufacturing, such as: coolant for electronic devices, solar energy, automotive generators, transformers, industry, and nuclear systems and so on [24] . In recent years, hybrid nanofluids have become the first choice over conventional fluids because of this several reasons. They possess properties such as electrical conductivity and permeability [25], they are more efficient at transferring heat [26], and Some of the combinations of two nanoparticles make the HNF moreluid become more stable [27]. Yahya *et al.*, [28] discussed the Runge-Kutta method was utilized to examine thermal dissipation, heat source, and magnetic field in ordinate engine oil using the novel hybridization of MoS_2+ZnO.The investigation of heat sources and their impact on heat transport is crucial in light of a wide variety of physical issues. Heat source effects on hybrid nanofluid using the headline method were documented by Alsabery*et et al.*, [29]. There are recent additions to the field that consider hybrid and conventional nanofluids with heat and mass transfer under a variety of physical conditions [30-39].

Recently, the subject of magnetic field for Newtonian and non–Newtonian fluid flows engrossed many researchers, because of its widespread applications in different fields. The magnetic field treated as an external agent to develop the thermal conductivity and thermophysical attributes of fluid flow. The flows consist of magnetic field mostly used in the problems which are related to geophysical and astrophysical mechanism, but recently magnetohydrodynamics flows are used in cancer treatment, fusion power, plasma studies, MRI, generators, heat exchanger process etc. Sarpakaya [40] first time surveyed the non-Newtonian fluid flow by means of magnetic field. Hussanan *et al.*, [41] studies the magnetohydrodynamic (MHD) flow of a Casson fluid and the heat transfer to a nonlinearly stretching sheet with Newtonian heating. Alkasasbeh [42] deliberated the mixed convective MHD flow of micropolar Casson fluid across a solid sphere. The effects of varying viscosity on the flow of axisymmetric nanofluids subjected to unsteady magnetohydrodynamics (MHD) with thermal diffusion were studied by Bagh *et al.*, [43]. Tlili *et al.*, [44] evaluated computational studies of MHD dissipative flow through a stretched sheet. The slip effects of MHD micropolar nanofluids were explored by Sohaib *et al.*, [45]. The effects of a heat source on MHD flow of nanofluid employing a stretched sheet were studied by Abbas *et al.*, [46].Some recent studies about MHD fluid flow are seen in the Refs [47-52].

In consideration of the afore-mentioned development and research applications in nanofluids, our key objective is to inspect in it in contemporary work, we have MHD of Williamson hybrid nanofluid of boundary layer stream over a stretching sheet in the company of thermal radiati on. The governing boundary layer equations are altered to nonlinear ODEs via similarity transformation which are formerly determined numerically using Keller box methods. The consequence of numerous parameters via velocity, temperature, and concentration distribution are inspected graphically. The skin friction and Nusselt number are also observed by tabulated data. Furthermore, a comparison of the present article has been presented with previous literature.

2. Mathematical Model

Here, we consider 2D, incompressible, free convection boundary layer flow of Williamson hybrid nanofluid by the influence of MHD and thermal radiation over stretching sheet. In the present investigation the boundary constant wall temperature are also taken. To the analysis of transport of mass and heat, the thermal radiation and thermophoretic impacts are considered. Two perpendicular coordinate systems (*x, y*) are chosen since sheet motion follows the x-axis and the wall is non-compressible (v_w = 0) (see in Figure 1). With the usage of boundary layer approximation and above assumption the emerging equations are stated as, Ref [18, 28, 30].

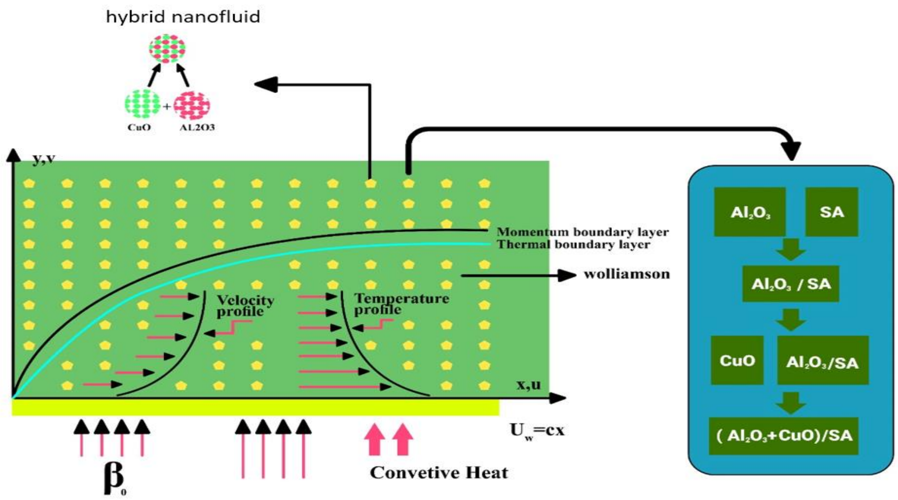

Fig. 1. Physical sketch of the model

Continuity equation

$$\frac{\partial u}{\partial \mathrm{x}} + \frac{\partial v}{\partial \mathrm{y}} = 0 \tag{1}$$

Momentum equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=V_{hnf}\frac{\partial u}{\partial y^2}-\frac{\sigma_{hnf}}{\rho_{hnf}}\beta_0^2u+\sqrt{2}\Omega v_f\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2} \tag{2}$$

Thermal energy equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_{hnf}}{(\rho C_p)_{hnf}}\frac{\partial^2 T}{\partial y^2}-\frac{1}{(\rho C_p)_{hnf}}\frac{\partial q_r}{\partial y} \tag{3}$$

For above equations boundary conditions are inscribed as:

$$\begin{aligned} &u=U_w=cx,\ v=0,\ T=T_w(x)\ at\ y=0\\ &u\to 0,\ T\to T_\infty\ as\ y\to\infty \end{aligned} \tag{4}$$

By considering Roseland approximation, the radiative heat flux at this point

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}; \tag{5}$$

Here σ^*, q_r, k^* are representative correspondingly as the Boltzmann constant, absorption coefficient and the radiative heat flux. Assuming a minor temperature alteration in flow, the Taylor series estimate for T_4 in expressions of T_∞ is as acquire

$$T^4\cong 4TT_\infty^3-3T_\infty^3 \tag{6}$$

$$\frac{\partial q_r}{\partial y}=\frac{16\sigma^*T_\infty^3}{3k^*v_f(\rho C_p)_f}\frac{\partial^2 T}{\partial y^2} \tag{7}$$

Table 1
Thermo-physical properties of hybrid nanofluid and nanoparticles

Properties	Nanofluid	Hybrid nanofluid
μ Viscosity	$\mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}}$	$\mu_{hnf}=\frac{\mu_f}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}}$
ρ Density	$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s$	$\rho_{hnf}=\left[(1-\phi_2)\{(1-\phi_1)\rho_f+\phi_1\rho_{s1}\}\right]+\phi_2\rho_{s2}$
ρC_P Heat capacity	$(\rho C_p)_{nf}=(1-\phi)(\rho C_p)_f+\phi(\rho C_p)_s$	$(\rho C_p)_{hnf}=\left[(\rho C_p)_f(1-\phi_2)\left((1-\phi_1)+\frac{\phi_1(\rho C_p)_{s1}}{(\rho C_p)_f}\right)+\phi_2(\rho C_p)_{s2}\right]$
K Thermal conductivity	$\frac{K_{nf}}{K_f}=\frac{K_s+(s_f-1)K_f-(s_f-1)\phi(K_f-K_s)}{K_s+(s_f-1)K_f+\phi(K_f-K_s)}$	$\frac{K_{hnf}}{K_f}=\frac{(K_{s2}+2K_{bf})-2\phi_2(K_{bf}-K_{s2})}{(K_{s2}+2K_{bf})+\phi_2(K_{bf}-K_{s2})}$ where $\frac{\kappa_{bf}}{\kappa_f}=\left[\frac{(\kappa_{s1}+2\kappa_f)-2\phi_1(\kappa_f-\kappa_{s1})}{(\kappa_{s1}+2\kappa_f)+\phi_1(\kappa_f-\kappa_{s1})}\right]$
σ Electrical conductivity	$\frac{\sigma_{nf}}{\sigma_f}=1+\frac{3(\sigma-1)\phi}{(\sigma+2)-(\sigma-1)\phi}$	$\frac{\sigma_{hnf}}{\sigma_{bf}}=\left[1+\frac{3\phi_1(\sigma_1\phi_1+\sigma_2\phi_2-\sigma_{bf}(\phi_1+\phi_2))}{(\sigma_1\phi_1+\sigma_2\phi_2+2\phi\sigma_{bf})-\phi\sigma_{bf}((\sigma_1\phi_1+\sigma_2\phi_2)-\sigma_{bf}(\phi_1+\phi_2))}\right]$

The variables of similarity for the problem are given as [18]

$$\psi = (cv)^{1/2} x f(\eta), \quad \eta = (cv)^{1/2} y, \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{8}$$

Here, stream function is defined in term of velocity components $u = \frac{\partial \psi}{\partial y}, \; v = -\frac{\partial \psi}{\partial x}$

By using similarity transformations, stream function and Table 1 in Eqs. (1)-(4) the PDEs are transformed into following ordinary equations:

$$f''' - A_1(f'^2 - ff'') - A_2 M f' + A_1 We f'' f''' = 0 \tag{9}$$

$$\theta'' \left[1 + \frac{1}{\mathrm{A}_4} \mathrm{Pr} N_r\right] + \frac{\mathrm{A}_3}{\mathrm{A}_4} \mathrm{Pr} f \theta' = 0 \tag{10}$$

The nondimensionalized form of the boundary conditions are,

$$\begin{aligned} &f(0) = 0, \quad f'(0) = 1, \; \theta(0) = 1 \text{ at } \eta = 0 \\ &\qquad\quad f'(\eta) \to 0, \; \theta(\eta) \to 0 \text{as } \; \eta \to \infty \end{aligned} \tag{11}$$

The dimensionless parameters that are used in above equations are stated as:

where $\mathrm{Pr} = \frac{v_{f(\rho C_p)f}}{k_f}$ the Prandtl number, $M = \frac{\sigma B_0^2}{\rho_{\mathrm{f}} c}$the magnetic field parameter, $N_r = \frac{16}{3} \frac{\sigma^*}{k^*} \frac{T_\infty^3}{v_f (\rho C_p)_f}$ the radiation parameter and $We = x\sqrt{2}\Omega \sqrt{\frac{c^3}{v_f}}$ the Weissenberg number

Also,

$$A_1 = (1 - \phi_1)^{2.5} (1 - \phi_2)^{2.5} \left[(1 - \phi_2)\left\{(1 - \phi_1) + \phi_1 \frac{\rho_{\mathrm{s1}}}{\rho_f}\right\} + \phi_2 \frac{\rho_{\mathrm{s2}}}{\rho_{\mathrm{f}}}\right]$$

$$A_2 = (1 - \phi_2)\left\{(1 - \phi_1) + \phi_1 \frac{\rho_{\mathrm{s1}}}{\rho_{\mathrm{f}}}\right\} + \phi_2 \frac{\rho_{\mathrm{s2}}}{\rho_{\mathrm{f}}}$$

$$A_3 = (1 - \phi_2)\left\{(1 - \phi_1) + \phi_1 \frac{(\rho C_p)_{p_1}}{(\rho C_p)_f}\right\} + \phi_2 \frac{(\rho C_p)_{p_2}}{(\rho C_p)_f},$$

$$A_4 = \left[\frac{(\kappa_{s_2} + 2\kappa_f) - 2\phi_2(\kappa_f - \kappa_{s_2})}{(\kappa_{s_2} + 2\kappa_f) + \phi_2(\kappa_f - \kappa_{p_2})}\right]\left[\frac{(\kappa_{s_1} + 2\kappa_f) + \phi_1(\kappa_f - \kappa_{s_1})}{(\kappa_{s_1} + 2\kappa_f) - 2\phi_1(\kappa_f - \kappa_{s_1})}\right]$$

$$A_4 = (1 - \phi_1)^{2.5} (1 - \phi_2)^{2.5}$$

The non-dimensional physical quantities in this problematic are the Nusselt numbers, *Nu* and skin friction coefficient *Cf* distinct as:(see Refs. [28, 42]):

$$Nu = \frac{x q_w}{k_f (T_w - T_\infty)}, Cf = \frac{\tau_w}{\rho_f U_w^2} \tag{12}$$

Here τ_w, q_w represents the wall shear stress and heat flux correspondingly expressed as:

$$q_w = k_{hnf}\frac{\partial T}{\partial y}, \tau_w = \mu_{hnf}\left(\frac{\partial u}{\partial y} + \frac{\Omega}{\sqrt{2}}\left(\frac{\partial u}{\partial y}\right)^2\right) at \quad y = 0 \tag{13}$$

Finally, by placing the value of τ_w and q_w in the previous equation of skin friction and Nusselt number, converted as:

$$C_f Re^{\frac{1}{2}} = \frac{1}{A_5}[f''(0) + Wef''(0)^2] \tag{14}$$

$$NuRe^{-\frac{1}{2}} = -\frac{k_{hnf}}{k_f}(1 + N_r)\theta'(0) \tag{15}$$

where $Re = \frac{xU_w}{v_f}$ denotes Reynolds number

3. Numerical Solution

A Keller-box method was first proposed by Keller and Bramble [53] in 1971. Jones [54] solved boundary layer problems using this method about a decade later and also solved boundary layer problems using this method about a decade later. Keller-box procedure is discussed in detail in Cebeci and Bradshaw [55]. During more than three decades, it proved to be effective and capable of constructing an accurate numerical solution to the issues related to boundary layers by being unconditionally stable, with second-order convergence. In the present study, a numerical solution was constructed using this method. The Keller-box method procedure is illustrated in the following inflow Figure 2:

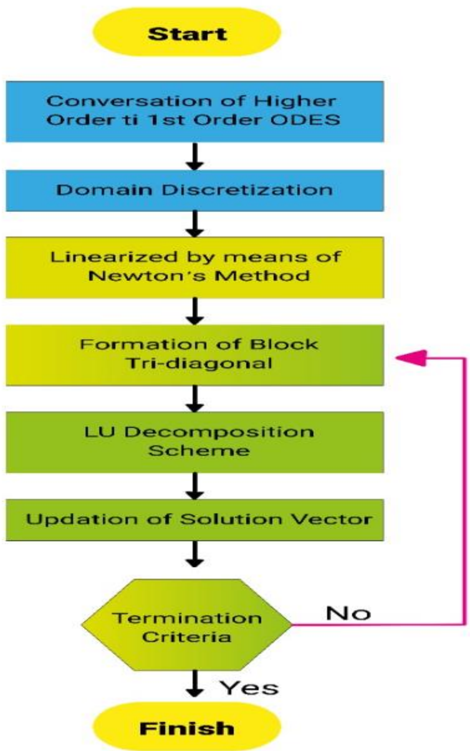

Fig. 2. Methodology of KBM

4. Results and Discussion

In this segment, we have discoursed the velocity and heat transmission features of Williamson hybrid nanofluid over a stretching sheet. The numerical problem is governed by physical parameters which comprise (M), (We), and (Nr). Solutions by KBM passed through MATLAB and calculated results are shown through graphs and tabular forms.

We selected Aluminium oxide (Al_2O_3) because of its great heat resistance and low electrical conductivity. In chemical reactions, copper oxide (CuO) is a popular oxidizing/reducing agent and process regulator. Also, sodium alginate (SA) was utilized as a thickening and moisture stabilizer.

Table 2
provides a comparison of their thermophysical qualities [19]

Thermo-Physical property	Al_2O_3	CuO	SA
$\rho(\mathrm{kg/m^3})$	6510	3970	989
$C_p(\mathrm{J/kgK})$	540	765	4175
$K(w/mK)$	18	40	0.6376
$\sigma(s/m)$	5.96×10^7	3.5×10^7	2.6×10^{-4}
Pr			6.5

Table 3 displays the numerical results, which are in remarkable agreement with the existing data. We compared our findings to those of Hassanien *et al.*, [56], Salah *et al.*, [57], and Alkasasbeh *et al.*, [18] to show that our suggested numerical approach for the Williamson hybrid nanofluid over a stretched sheet is valid, accurate, and precise. This strengthens our conviction that the findings presented in this paper are very good.

Table 3
Comparison of $Re^{-1/2}\,Nu$ with variation in Prandtl number, when M=We= Nr=$\phi_1=\phi_2=0$

Pr	$Re^{-1/2}\,Nu$			
	Hassanien *et al.*, [56]	Salleh *et al.*, [57]	Alkasasbeh *et al.*, [18]	Present
0.72	0.46325	0.46317	0.46316	0.46357
1	0.58198	0.58198	0.58198	0.58198
3	1.16525	1.16522	1.16524	1.16524
5		1.56806	1.56807	1.56806
7		1.89548	1.89550	1.89551
10	2.30801	2.30821	2.30820	2.30821
100	7.74925	7.76249	7.76250	7.76250

The numerical results of Nusselt number and the friction coefficient for various values of magnetic field parameter M, the radiation parameter Nr and the Weissenberg number We are compiled in Table 4, which shows what happens when the hybrid nanofluid and the nanofluid are exposed to the same circumstances. Both the Nusselt number and the friction coefficient of the hybrid nanofluid decrease when M is decreased throughout this study. Compared to nanofluid, the melting point of the hybrid nanofluid is lower. This drop is also not usual for most fluids, so it's notable in both cases. Everyone knows that raising the Nusselt number of a fluid makes it less thick and more fluid. When compared to the influence of the (We) coefficient, Nusselt numbers exhibits a considerable rise when (We) are elevated, in the coefficient of friction addition to a very minor change in both fluids. This is in contrast to the effect of the M coefficient. The effect of Nr on Nusselt number in both fluids shows that the values are stable, which shows that its effect is limited other than at low the coefficient of friction, where there seems to be little change in both fluids.

The effect of magnetic field parameter M on the velocity component f′(η) and temperature θ(η) are seen in Figure 3 and Figure 4, respectively for both Al_2O_3+CuO/SA hybrid nanofluid and Al_2O_3/SA nanofluid. When a magnetic field is generated, the Lorentz force appears works to hinder the flow of the fluid, and its presence causes a rapid reduction in velocity at the boundary. In contrast, as fluid particles move, lines of magnetic field are distorted and electric current is produced, which is converted into heat energy due to the resistance of the fluid particles to electric current flow. This heat is utilized to increase the total energy of the fluid particles and therefore, temperature rises and the velocity of fluid is reduced.

Table 4
Values of $\mathrm{Re}^{-1/2}Cf$ and $Re^{-1/2}\,N\,u$ for different values of *We, Nr*, and *M*

			Hybrid nanofluid (Al_2O_3+CuO)/SA		Nanofluid (Al_2O_3)/SA	
M	We	Nr	$Re^{-1/2}Nu$	$Re^{-1/2}Cf$	$Re^{-1/2}Nu$	$Re^{-1/2}Cf$
1	0.1	2	0.74280	-2.14012	1.15985	-1.95551
2			0.63030	-2.73888	1.00399	-2.45209
3			0.57294	-3.14333	0.91738	-2.81242
2	0.01	2	0.63375	-3.29974	1.01110	-2.84748
	0.1		0.63030	-2.73888	1.00399	-2.45209
	0.5		0.61270	-0.56004	0.96532	-0.17412
2	0.1	1	0.62859	-2.73888	0.99239	-2.45209
		2	0.55091	-2.73888	0.90031	-2.45209
		3	0.54284	-2.73888	0.89042	-2.45209

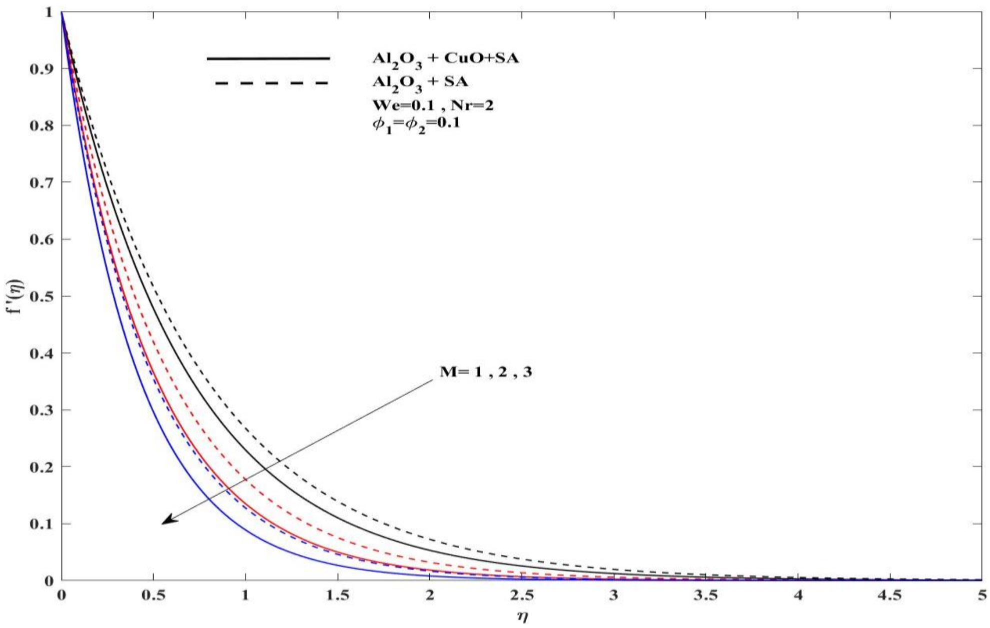

Fig. 3. Velocity f′(η) fluctuation with M

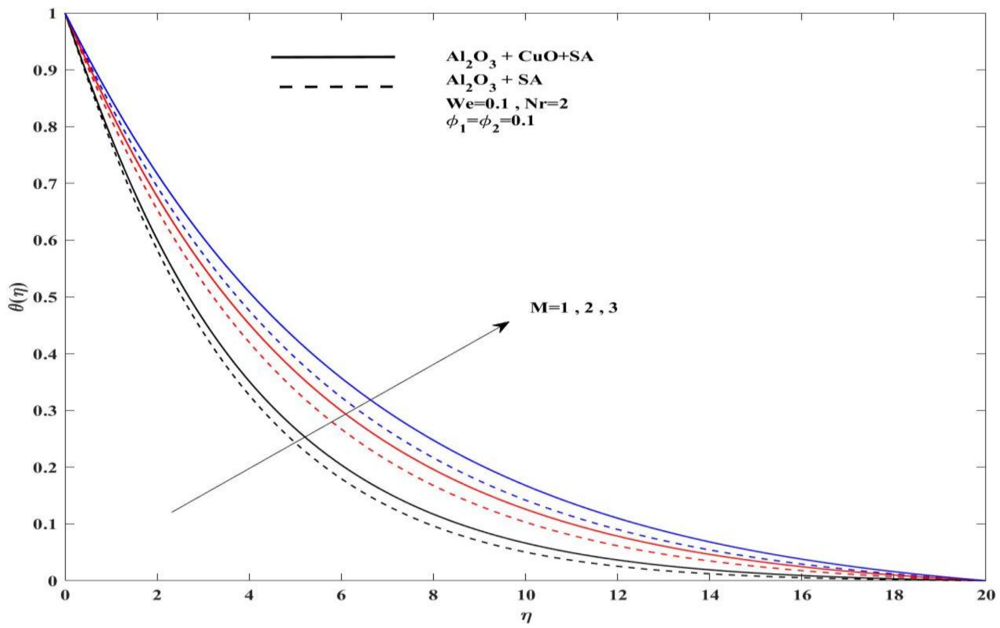

Fig. 4. Temperature θ(η) fluctuation with M

Figures 5 and 6 illustrate the influence of Weissenberg number (We) on velocity f' (η) and temperature for both hybrid nanofluid and nanofluid. It appears that the velocity of both of the fluids decreases with the increasing values of (We) and the opposite happens with respect to temperature. In order to comprehend stretching processes like natural shear, it is vital to know the Weissenberg number (We), which specifies the degree of deformation anisotropy or direction. When subjected to frictional manipulations, the ephemeral fluid's non-Newtonian property is amplified, leading to increased fluidity. As a result of the fluid's reduced sensitivity to shear, less of the momentum of the moving boundary wall is transmitted to the fluid. Because of this, fluidity is hindered when fluid motion slows and the thickness of the momentum barrier layer lowers.

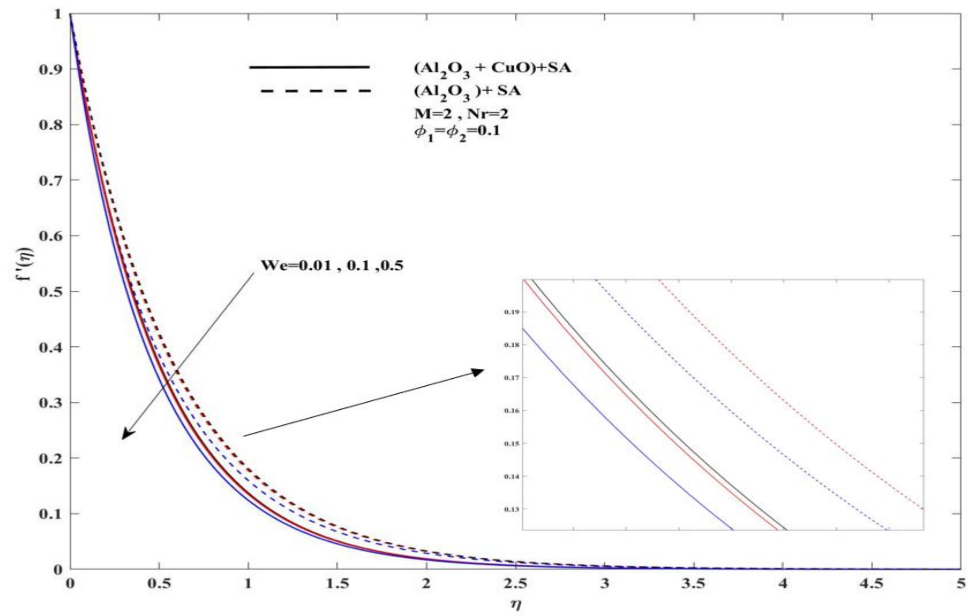

Fig. 5. Velocity f'(η) fluctuation with We

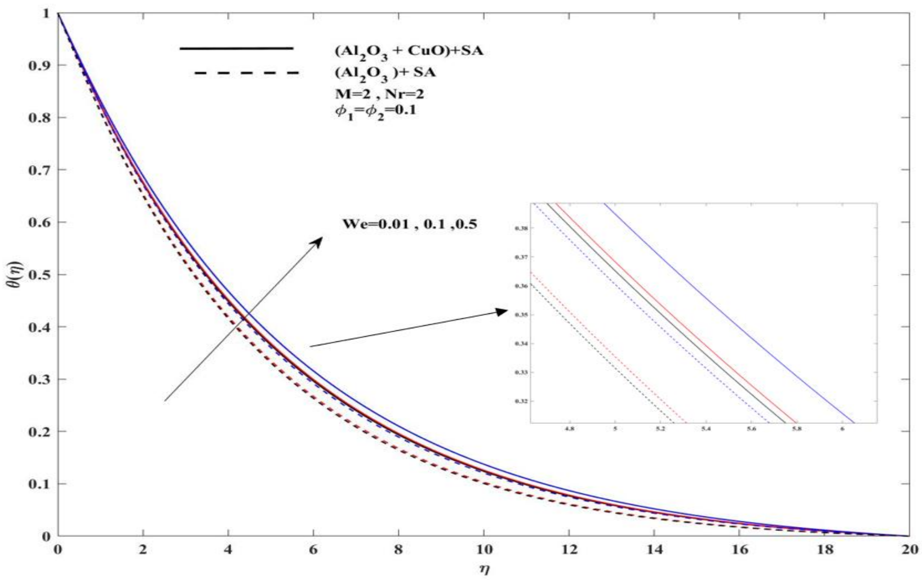

Fig. 6. Temperature θ(η) fluctuation with We

Finally, the temperature is shown for different thermal radiation intensities Nr in Figure 7 for both Al_2O_3+CuO/SA hybrid nanofluid and Al_2O_3/SA nanofluid. The radiation parameter is used to quantify the connection between the two forms of heat transport (conduction and thermal radiation). As a result, a higher value for Nr indicates that more radiative heat energy is being introduced into the system, leading to a higher temperature. Consequently, the radiation may regulate the temperature of the boundary layers.

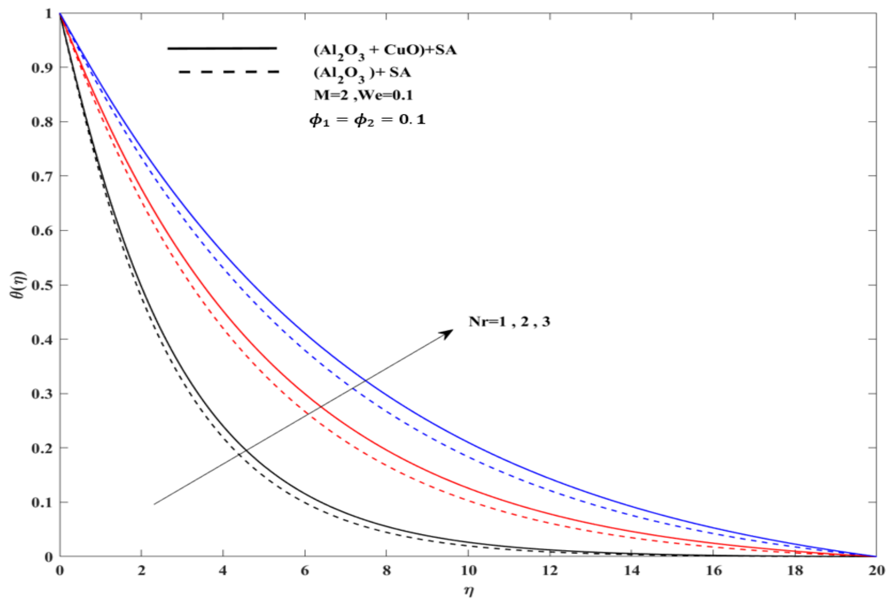

Fig. 7. Temperature θ(η) fluctuation with Nr

5. Conclusion

In this work, we analysed the consequences of heat and mass transfer of magnetic Williamson hybrid nanofluid flow over a stretched sheet with thermal radiation. The existing flow equations are tackled by KBM passed through MATLAB. The key results of the present analysis are :

i. Increasing M and We contributions cause f'(η) to decrease dramatically for both fluids.
ii. A rise in temperature θ(η) due to an increase in M, Nr, and We.
iii. Increases in We, Nr and M lead to a lower skin friction coefficient *Cf*.
iv. The Nusselt number *Nu* outlines for both fluids reduce with rise in M, and the opposite happens with the effect of We.
v. The velocity f'(η) fluctuation of a fluid is not significantly affected by the Nr parameter.
vi. Williamson nanofluid has the highest value for Nusselt number, skin friction and velocity profile. Moreover, it is the lowest value for temperature profile.

Acknowledgments
The authors heartily thank the Deanship of Scientific Research and Postgraduate Studies, Al-Hussein Bin Talal University, Ma'an—Jordan, for the financial support through voting No. (399/2020) for this research.

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