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Study of the MHD Flow of Casson Nanofluid in the Presence of Oxides Nanoparticles Based $C_2H_6O_2/H_2O$ Under Constant Heat Flux Boundary Condition

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Abstract – *In this study, a numerical investigation of free convection in magneto-hydrodynamic (MHD) Casson nanofluid flow with heat transfer over a stretching sheet and with constant heat flux boundary condition is done. A magnetic field is considered normal to the stretching sheet. Two types of nanoparticles with various conductivities, such as iron oxide and graphene oxide are suspended in ethylene glycol/water-based Casson nanofluid. An implicit finite difference scheme known as Keller-box method has been employed in order to solve the set of nonlinear partial differential equations resulted from the governing equations, namely continuity, momentum, and energy equations. The effects of the MHD Casson nanofluids parameters on the physical properties such that local skin friction coefficient, local Nusselt number, velocity, and temperature have been displayed and discussed. The study has revealed that ethylene glycol has higher velocity and less temperature than water with different values of nanoparticle volume fraction, magnetic, and Casson parameters with the existence of Fe_3O_4/GO nanoparticles. Comparisons with previous studies show that the proposed method is in good agreement with other results.* **Copyright © 2021 Praise Worthy Prize S.r.l. - All rights reserved.**

***Keywords**: Casson Fluid, Nanofluid, Magneto-Hydrodynamic, Stretching Sheet, Ethylene Glycol*

Nomenclature

a	Constant
B_0	Magnetic field strength
C_f	Local skin friction coefficient
k_s	Thermal conductivity of nanoparticles
k_f	Thermal conductivity of base fluid
k_{nf}	Thermal conductivity of nanofluid
M	Magnetic parameter
Nu	Local Nusselt number
Pr	Prandtl parameter
p_y	Yield stress of based fluid
q_w	Wall heat flux
T	Temperature of the fluid
T_w	Wall temperature
T_∞	Ambient temperature
u	x-component of velocity
v	y-component of velocity
v_f	Kinetic viscosity of base fluid
v_{nf}	Kinetic viscosity of nanofluid
α	Thermal diffusivity
α_{nf}	Thermal diffusivity of nanofluid
β	Casson parameter
θ	Dimensionless temperature
f	Dimensionless velocity
Re	Local Reynolds number
u_∞	Stream velocity
u_s	Straining velocity
u_w	Stretching velocity
x, y	Cartesian coordinates
π	Product the component of the deformation rate with itself
π_c	Critical value of the product
η	Dimensionless similarity variable
σ	Electrical conductivity
χ	Nanoparticle volume fraction
τ_w	Shear stress
μ_f	Dynamic viscosity of the base fluid
μ_{nf}	Dynamic viscosity of the base nanofluid
ρ_f	Density of base fluid
ρ_{nf}	Density of base nanofluid
$(\rho C_p)_f$	Heat capacity of base fluid
$(\rho C_p)_{nf}$	Heat capacity of base nanofluid
μ_B	Plastic dynamic viscosity of the fluid

I. Introduction

The normal base fluids, such as ethylene glycol, water, and kerosene oil have low heat transfer properties.

Moreover, heat transfer development is considered to be substantial subject in terms of the thermal conductivity boost among researchers [1]. For this reason, nanoparticles suspended in the base fluid have been investigated. For instance, Choi and Eastman [2] have reported the new category of heat transfer fluids by suspending the nanoparticles in conventional heat

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transfer fluids. In addition, Buongiorno [3] and Tiwari and Das [4] have presented different mathematical models that have been applied extensively in order to study the characteristics of nanofluids. These models have studied the convection boundary layer flow in nanofluid over many geometries such as stretching sheet, solid sphere, circular cylinder see: Tiwari and Das [4] model, Hussanan et al. [5], Swalmeh et al. [6], Abu-Nada et al. [7], Alwawi et al. [8], Hayat et al. [9] and Abbas et al. [10]. The unsteady flow and convection heat transfer at the stagnation point on a stretching/shrinking sheet in the presence of a porous medium filled with a hybrid nanofluid has been considered by Aly and Pop [11].

Tarakaramu and Satya Narayana [12] have investigated the numerical study for Chemical Reaction impacts on Bio-Convection Nanofluid flow between two Parallel Plates in a Rotating System with Variable Viscosity. In addition, the numerical solutions of the convection boundary layer flow in micropolar nanofluid over solid sphere and the horizontal circular cylinder have been studied in Qadan et al. [13], Alzgool et al. [14] and Swalmeh et al. [15]. It is well known that the viscosity and flow demeanor for the biological and industrial fluids are changed with the impact of stress and hence its viscosity characteristic deviates from the Newtonian fluid law. Various models of non-Newtonian fluids based on their several flow impacts have been suggested by many studies. The most remarkable non-Newtonian fluid are Casson fluid, micropolar fluid, Maxwell Viscoelastic fluid, Euyring-Powell fluid, and Walters-B fluid. In this study, Casson fluid has been considered. Casson theory has been firstly introduced by Casson [16]. Casson fluid is a shear-thinning fluid with infinitely much viscosity at zero rates of shear, yield stress below which no flow occurs, and a zero viscosity at infinitely many rates of shear, Pramanik [17]. Some famous examples of the Casson fluids are as follows: shampoos, toothpaste, jellies, tomato, kinds of honey, soups, concentrated fruit juices, etc. Ethylene glycol can also be considered as a Casson fluid. Research interested on convection and heat transfer boundary layer flow in the presence of Casson fluid on stretching sheet can be referred to Mustafa et al. [18], Hayat et al. [19], Mukhopadhyay et al. [20], Qing et al. [21] and Alkasasbeh et al. [22]. The study of gyro-tactic microorganism and nanoparticles in the bio-convection MHD flow of Casson fluid at the nonlinear stretching boundary has been conducted by Ansari et al. [23]. On the other hand, the numerical studies of the free magneto-hydrodynamic (MHD) convection boundary layer flow, in presence of magnetic field, have taken a vast space in fluid dynamics researches. MHD free convection boundary layer flow in nanofluid on stretching sheet with thermal radiation and magnetic field has been investigated by Tarakaramu and Satya Narayana [12]. Ishak [24] has studied the influences of radiation on MHD boundary layer flow over an exponentially stretching sheet. The author has obtained the required ordinary differential equations from the governing system of partial differential equations, and then, has solved the corresponding equation numerically using an implicit finite-difference technique. The finding from this study shows that the rate of local heat transfer, at the surface, decreases with the increase of values from the radiation and magnetic parameters. A study by Ghadikolaei et al. [25] has investigated the heat transfer and MHD boundary layer flow of some nanoparticles which include an incompressible Iron (II,III) oxide (Fe_3O_4)-ethylene glycol ($(CH_2OH)_2$) on micropolar fluid the presence of Joule heating and thermal radiation. The research has derived an Ordinary Differential Equation (ODE) from the transformed Partial Differential Equations (PDEs) using the Runge–Kutta Fehlberg fourth fifth order technique. Their findings show that the reduction of velocity for both the dust and the fluid phases happens as a result of Lorentz force against flow by the increase of the number of Hartman, temperature enhancement and thermal boundary layer thickness. In addition, Abro et al. [26] have investigated the influence of the nanoparticle's magnetite $Fe_3O_4Fe_3O_4$ on free convection flow of nanofluid with MHD. These nanoparticles have been considered as a conventional base after being dispersed in water. The authors have fractionized the governing equations using Caputo–Fabrizio and Atangana–Baleanu fractional operators in order to analyze the new fractional derivatives. Through Caputo–Fabrizio and Atangana–Baleanu fractional, the researchers have presented the graphical comparison for four types of models, which include ordinary nanofluid without magnetic field on fluid flows, ordinary nanofluid with magnetic field, fractionalized nanofluid without magnetic field, and fractionalized nanofluid with magnetic field. It is further noted that a new approach has been found out in Alamri et al. [27], for Cattaneo-Christov heat flux model using the influence of mass transfer on MHD second grade fluid towards stretching cylinder. Moreover, Alwawi et al. [28] have found Sodium Alginate based Casson nanofluid natural convection flow about the solid sphere with constant surface heat flux in the presence of a magnetic field, Alwawi et al. [8]. [29] and [30] are considered to be novel and efficient studies. This study considers the effects of some parameters such as nanoparticle volume fraction, Casson, and magnetic parameters, the constant heat flux boundary condition. The mathematical formulations of PDE's for the governing equations of this problem are solved via Keller Box method.

The rest of the paper is structured as follows. In Section II, the mathematical model for the problem of free convection in MHD Casson Nanofluid is presented and the steps of converting the dimensional governing equations of the model to non-dimensional governing equations are illustrated. In addition, this section shows the transformation process of the non-dimensional governing equations to the PDEs. The result of numerical experiment based on velocity, temperature profiles, local skin friction, and Nusselt number is presented in Section III. Finally, the conclusion is discussed in Section IV.

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II. Mathematical Formulation

The problem of Magneto-hydrodynamic (MHD) steady free convection boundary layer flow of two kinds of oxides nanoparticles (Fe_3O_4, GO) in a steward Casson fluid on a stretching sheet with a constant heat flux q_w, is taken into account. Ethylene glycol ($C_2H_6O_2$) and water (H_2O) are considered as based fluid in this study. Figure 1 shows the physical demonstration of this problem, where, the flow has started at $y = 0$ (O), and has been limited in $y = \infty$. u_s and u_w are called shrinking/stretching velocity and straining velocity, respectively. On the other hand, the flow is assumed to be in the x-orientation, which means that the flow is extended along the stretching sheet in the upward orientation, and the y-axis is orthogonal to it. At the same time, the nanofluid is diffused in a Casson environment and a variable magnetic field strength B_0, which is applied in the fluid flow cross-orientation. T_∞ is the ambient temperature of the Casson nanofluid. The following equation is obtained to determine the rheological property of the Casson fluid (Casson [16]):

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + p_y / \sqrt{2\pi}\right) e_{ij}, & \pi > \pi_c \\ 2\left(\mu_B + p_y / \sqrt{2\pi_c}\right) e_{ij}, & \pi > \pi_c \end{cases} \tag{1}$$

Here, $\pi = e_{ij} e_{ij}$, where, e_{ij} is the (i,j)-th component of the deformation rate, π_c, p_y, and μ_B are respectively, the critical values of this product based on the non-Newtonian model, the yield stress of the fluid, and the plastic dynamic viscosity of the non-Newtonian fluid.

In the case of the Casson fluid, $\pi > \pi_c$, where:

$$\mu = \mu_B + p_y / \sqrt{2\pi} \tag{2}$$

The value of $p_y = \mu_B \sqrt{2\pi} / \beta$ is substituted in equation (5), then the kinematic viscosity of the Casson fluid, μ_B depends on the plastic dynamic viscosity, ρ is the density, and β is the parameter of the Casson fluid, where:

$$\mu / \rho = \left(\mu_B / \rho\right)\left(1 + \frac{1}{\beta}\right) \tag{3}$$

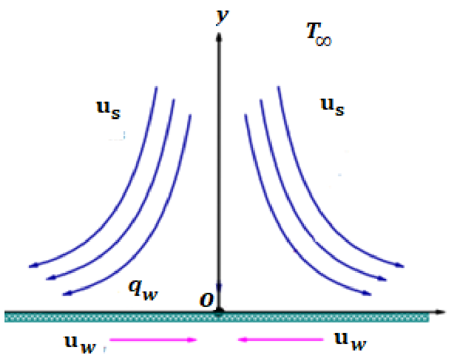

Fig. 1. Physical demonstration of the problem

On the other hand, the magnetic field term in the momentum equation is defined as $\left(\sigma_{nf} / \rho_{nf}\right) B_0^2 \breve{u}$ as in Alwawi et al. [28]. Now, the continuity, the momentum, and the thermal energy in dimensional form for the convection boundary layer flow on the stretching sheet in Casson nanofluid can be expressed as follows (Salleh et al. [31] and Tiwari and Das [4]):

$$\frac{\partial(\breve{u})}{\partial \breve{x}} + \frac{\partial(\breve{v})}{\partial \breve{y}} = 0 \tag{4}$$

$$\rho_{nf}\left(\breve{u}\frac{\partial \breve{u}}{\partial \breve{x}} + \breve{v}\frac{\partial \breve{u}}{\partial \breve{y}}\right) = v_{nf}\left(1 + \frac{1}{\beta}\right)\left(\frac{\partial^2 \breve{u}}{\partial \breve{y}^2}\right) - \frac{\sigma_{nf}}{\rho_{nf}} B_0^2 \breve{u} \tag{5}$$

$$\breve{u}\frac{\partial T}{\partial \breve{x}} + \breve{v}\frac{\partial T}{\partial \breve{y}} = \frac{k_{nf}}{\left(\rho C_p\right)_{nf}}\left(\frac{\partial^2 \breve{T}}{\partial \breve{y}^2}\right) \tag{6}$$

The properties of nanofluid are expressed by Ahmed and Khan [32] and Swalmeh et al. [6] as follows:

$$\begin{aligned} (\mu)_{nf} &= \frac{\mu^f}{(1-\gamma)^{2.5}} \\ \left(\rho C_p\right)_{nf} &= \left(\gamma\left(\rho C_p\right)_s + (1-\gamma)\left(\rho C_p\right)_f\right) \\ (\alpha)_{nf} &= \frac{k_{nf}}{\left(\rho C_p\right)_{nf}} \\ (\rho)_{nf} &= \left(\gamma(\rho)_s + (1-\gamma)(\rho)_f\right) \\ \frac{k_{nf}}{k_f} &= \frac{\left(k_s + 2k_f\right) - 2\gamma\left(k_f - k_s\right)}{\left(k_s + 2k_f\right) + \gamma\left(k_f - k_s\right)} \\ \sigma_{nf} &= 1 + \frac{3\left(\left(\sigma_s / \sigma_f\right) - 1\right)\chi}{\left(\left(\sigma_s / \sigma_f\right) + 2\right) - \left(\left(\sigma_s / \sigma_f\right) - 1\right)\gamma} \end{aligned} \tag{7}$$

where the dimensional boundary condition is the constant surface heat flux (CHF), corresponding to the equations for free convection can be introduced as Salleh et al. [31], where:

$$\begin{aligned} &\breve{u} = u_w(\breve{x}) = a\breve{x}, \quad \breve{v} = 0, \quad \frac{\partial T}{\partial \breve{y}} = -\left(-\frac{q_w}{k_f}\right) \text{ as } \breve{y} = 0, \\ &\breve{u} \to 0, \quad \mathrm{T} \to \mathrm{T}_\infty, \quad \text{as } \breve{y} \to \infty \end{aligned} \tag{8}$$

The following similarity transformations with the dimensionless variables are used to convert the dimensional equations. The governing equations (4) to (6) are subject to the boundary conditions (8), as Salleh et al. [33]:

$$\begin{aligned} &\breve{u} = \frac{\partial \psi}{\partial \breve{y}}, \text{ and } \breve{v} = -\frac{\partial \psi}{\partial \breve{x}}, \quad \psi = \left(a / v_f\right)^{1/2} \breve{x} f(\eta), \\ &\eta = \left(a / v_f\right)^{1/2} \breve{y}, \theta(\eta) = \frac{k_f}{q_w}\left(T - T_\infty\right)\left(a / v_f\right) \end{aligned} \tag{9}$$

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which satisfies the continuity equation. Here, ψ is the stream function, and q_w is the surface heat flux. In conclusion, the following PDEs are obtained as shown below:

$$\left(\frac{\rho_f}{\rho_{nf}}\frac{1}{(1-\gamma)^{2.5}}\right)\left(1+\frac{1}{\beta}\right)f'''+ff''-(f')^2+ -\frac{\rho_f}{\rho_{nf}}\frac{\sigma_f}{\sigma_{nf}}Mf'=0 \quad (10)$$

$$\frac{1}{\Pr}\left(\frac{k_{nf}/k_f}{(1-\gamma)+\gamma(\rho C_p)_s/(\rho C_p)_f}\right)\theta''+f\theta=0 \quad (11)$$

subject to the following boundary conditions:

$$f'=1,\ f=0,\ \theta'=-1,\ \text{as } \eta=0, \quad f'\to 0,\ \theta\to 0,\ \eta\to\infty \quad (12)$$

Here $\frac{1}{\Pr}=\frac{v_f}{\alpha_f}$ is the Prandtl number and $M=\frac{\sigma_f}{a\rho_f}B_0^2$ is the magnetic parameter. The physical quantities such as the local skin friction coefficient C_f and the Nusselt number Nu, are given by:

$$C_f=\left(\frac{\tau_w}{\rho u_w^2}\right)_{\breve{y}=0} \text{ and } \mathrm{Nu}=\left(\frac{xq_w}{k_f(T_w-T_\infty)}\right)_{\breve{y}=0} \quad (13)$$

where, τ_w and q_w are corresponding to the, shear stress, and the heat flux on the plane of the wall and they are defined as:

$$\tau_w=\mu_{nf}\left(\frac{\partial\breve{u}}{\partial\breve{y}}\right)_{\breve{y}=0} \text{ and } q_w=-k_{nf}\left(\frac{\partial T}{\partial\breve{y}}\right)_{\breve{y}=0} \quad (14)$$

Now, the local skin friction coefficient and the Nusselt number are respectively:

$$\mathrm{Re}^{1/2}C_f=\frac{1}{(1-\gamma)^{2.5}}\left(1+\frac{1}{\beta}\right)f''(0), \quad \mathrm{Re}^{-1/2}\,\mathrm{Nu}=\left(\frac{k_{nf}}{k_f}\right)\frac{1}{\theta(0)} \quad (15)$$

where $\mathrm{Re}=\left(a\breve{x}^2/v_f\right)$ denotes the local Reynolds number.

III. Results and Discussion

In this paper, the numerical results for the free convection in MHD Casson nanofluid are discussed, with constant surface heat flux (CHF) boundary condition.

Here, iron oxide and graphene oxide suspended in water/ethylene glycol Casson nanofluid are considered.

The research has further studied the effect of different values of nanoparticle volume fraction, Casson, and magnetic parameters on temperature and velocity profiles, as well as local Nusselt number and local skin friction.

On the other hand, the Thermo-physical properties of used based fluids and nanoparticles are displayed in Table I. Comparing the findings to previous results, such as Elbashbeshy [34] and Salleh et al. (2010), gives a very good agreement as shown in Table II. The behavior of the physical quantities, such as local Nusselt number and local skin friction coefficient, under the impact of the nanoparticle volume fraction, magnetic, and Casson parameters are numerically obtained by Keller box method, as displayed in Table III. It is observed that when the Casson and the magnetic parameters increase, the local skin friction coefficient along with the local Nusselt number in the existence of Oxides nanoparticles suspended in water and ethylene glycol-based fluids decrease.

On the other hand, when the nanoparticle volume fraction increases, it decreases the local skin friction and increases the local Nusselt number.

Besides, it has been noticed from this table that graphene oxide-water/ethylene glycol has a high C_f and *Nu* compared with iron oxide-water/ethylene glycol Casson nanofluid. In addition, ethylene glycol has higher local skin friction coefficient and local Nusselt number than water.

The difference in velocity and temperature for various values of the nanoparticle volume fraction χ, Magnetic M, and Casson β parameters, are presented in Figures 2 to 7.

It can be noticed that the velocity of the Casson nanofluid decreases with increasing values of χ.

TABLE I
DIFFERENT VALUES OF THERMOPHYSICAL PROPERTIES OF OXIDES OF TWO BASE CASSON NANOFLUIDS (ALWAWI ET AL. [29], SWALMEH ET AL. [35], HAYATI ET AL. [36])

Physical properties	Water	Ethylene glycol	GO	Fe_3O_4
k (W/m K)	0.613	0.253	5000	9.7
ρ (kg/m^3)	997.1	1115	1800	5180
ρc_p (J/kg K)	4179	2430	717	670
σ(Sm^{-1})	5.5×10^{-6}	10.7×10^{-5}	1.1×10^{-5}	2.5×10^{4}
Pr	6.2	195	…	…

TABLE II
COMPARISON OF $\mathrm{Re}^{-1/2}\mathrm{Nu}$ WITH VISCOUS NEWTONIAN FLUID ($M = 0$, $\beta\to\infty$, AND χ=0), WITH SEVERAL VALUES OF *Pr*

Pr	Elbashbeshy [33]		Salleh et al. [32]		Present	
	$\mathrm{Re}^{-1/2}\mathrm{Nu}$	$\theta(0)$	$\mathrm{Re}^{-1/2}\mathrm{Nu}$	$\theta(0)$	$\mathrm{Re}^{-1/2}\mathrm{Nu}$	$\theta(0)$
0.72	0.46780	2.13767	0.46317	2.15902	0.46316	2.1591
1	0.58210	1.71792	0.58210	1.71828	0.58198	1.7183
3	1.16525		1.16522	0.85817	1.1653	0.85818
5	…	…	…	0.63770	1.5681	0.63772
7	…	…	…	0.52755	1.8955	0.52756
10	2.30730	0.43341	2.30728	0.43322	2.3082	0.43323
100	…	…	…	0.12851	7.7809	0.12852

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TABLE III
VALUES OF THE SKIN FRICTION COEFFICIENT $Re^{1/2}C_f$ AND LOCAL NUSSELT NUMBER $Re^{-1/2}Nu$ FOR DIFFERENT VALUES OF CASSON PARAMETER β, χ AND M

			Fe_3O_4 Water		GO Water	
β	χ	M	$Re^{1/2}C_f$	$Re^{-1/2}Nu$	$Re^{1/2}C_f$	$Re^{-1/2}Nu$
3	0.1	5	-15.749	2.1484	-14.218	2.4051
7			-34.024	2.0857	-30.711	2.3439
10			-47.685	2.0696	-43.045	2.328
3	0.075	5	-13.524	1.9754	-12.481	2.152
	0.15		-21.303	2.5379	-18.507	2.9949
	0.2		-28.773	2.9957	-24.23	3.7223
3	0.15	1	-10.849	3.0661	-9.8556	3.4719
		2	-14.202	2.8922	-12.589	3.3194
		3	-16.903	2.7542	-14.826	3.1954
		7	-24.934	2.369	-21.568	2.8338
			Fe_3O_4 Ethylene glycol		GO Ethylene glycol	
β	χ	M	$Re^{1/2}C_f$	$Re^{-1/2}Nu$	$Re^{1/2}C_f$	$Re^{-1/2}Nu$
3	0.1	2	-10.498	16.797	-9.7476	17.358
7			-22.683	16.749	-21.053	17.313
10			-31.791	16.737	-25.283	17.306
3	0.075	5	-13.348	14.861	-12.441	15.262
	0.15		-20.836	20.341	-18.391	21.413
	0.2		-28.028	24.992	-24.037	26.732
3	0.15	1	-10.672	20.907	-9.8273	21.905
		2	-13.927	20.726	-12.529	21.749
		3	-16.554	20.58	-14.744	21.622
		7	-24.377	20.144	-21.425	21.238

Besides, the Go-ethylene glycol/water has a higher velocity than Fe_3O_4-ethylene glycol/water. At the same time, the flow of ethylene glycol has higher velocity compared to water, with increasing the values of χ, as shown in Figure 2. It is clear from Figure 3, that an increase in nanoparticle volume fraction parameter χ decreases the temperature of the Casson nanofluid. It is also clear that there is a rise in Fe_3O_4 temperature compared to the GO nanoparticles, with an increase of χ.

On the other hand, it can be noted that the variations in ethylene glycol temperature are high compared to water. It is observed from Figure 4 that the fluid velocity decreases with increasing the magnetic parameter. In addition, the water has a lower velocity than ethylene glycol. In addition, GO nanoparticles have a higher velocity than Fe_3O_4. An increase in magnetic M implies an increase in the temperature of the Casson nanofluid, as plotted in Figure 5. As shown in this figure, the temperature of the Fe_3O_4 nanoparticles is higher than GO nanoparticle's temperature. It is also observed that the ethylene glycol has a lower temperature than water.

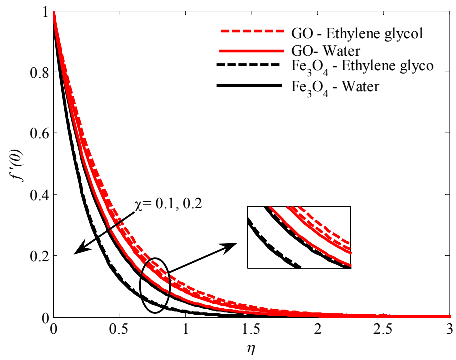

Fig. 2. Velocity profile vs. χ

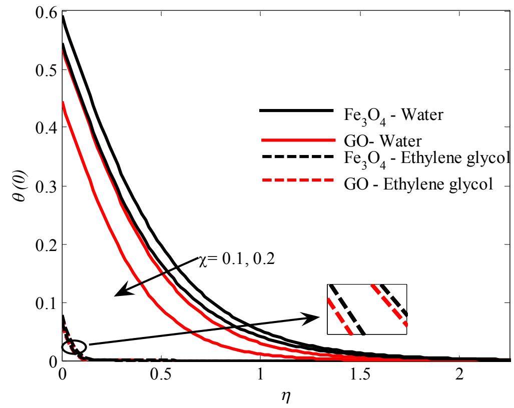

Fig. 3. Temperature profile vs. χ

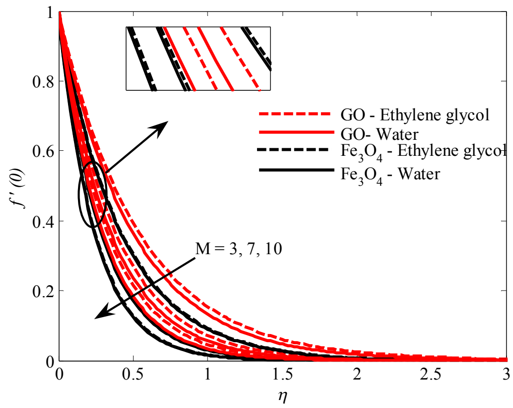

Fig. 4. Velocity profile vs. M

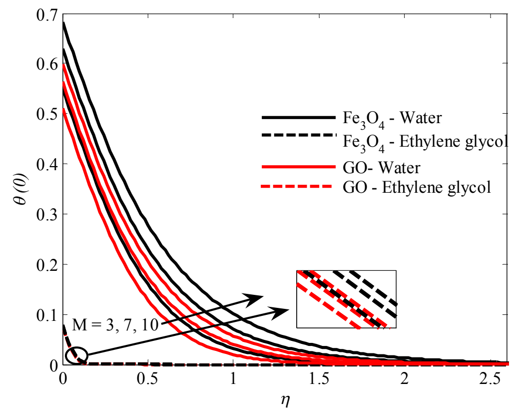

Fig. 5. Temperature profile vs. M

The effects of the different values of Casson parameter such as velocity and temperature profiles are shown in Figures 6 and 7, respectively.

An increase in Casson parameter reduces the Casson nanofluid velocity and increases the temperature. From these figures, it can be deduced that the ethylene glycol velocity is higher than water.

Meanwhile, the opposite case happens, where the water temperature flow is higher than ethylene glycol, with increasing the values of β. In addition, Fe_3O_4-ethylene/water has less velocity and higher temperature than GO-ethylene/water.

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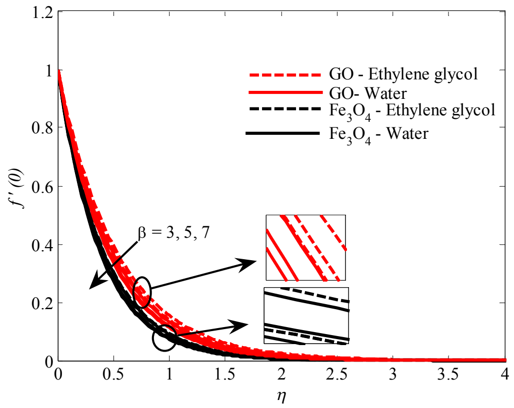

Fig. 6. Velocity profile vs. β

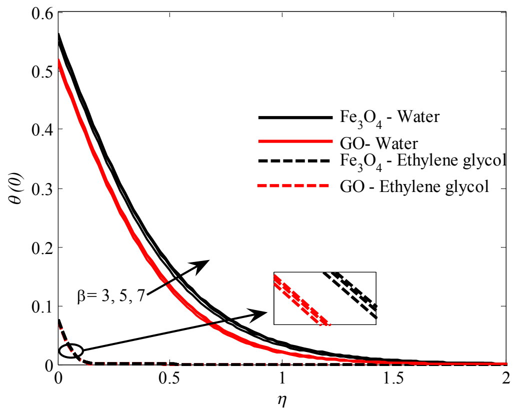

Fig. 7. Temperature profile vs. β

IV. Conclusion

The problem of free convection boundary layer flow on a stretching sheet is numerically discussed in this paper. The effects of the nanoparticle volume fraction, magnetic, and Casson parameters affect the values of the local Nusselt number, and local skin friction coefficient, as well as temperature and velocity profiles have been discussed. For constant surface heat flux boundary condition case, it can be concluded that the ethylene glycol has higher velocity and less temperature than water with different values of the nanoparticle volume fraction, magnetic, and Casson parameters. Besides, the graphene oxide velocity is higher than iron oxide velocity, and the converse case happens in temperature profile, the graphene oxide temperature is less than iron oxide temperature. In addition, when the nanoparticle volume fraction parameter is increased, the velocity, and the temperature profiles are decreased. On the other hand, an increase in Casson and magnetic parameters leads to a rise in the temperature and a decrease in the values of the velocity profile. The local skin friction and local Nusslt number, as well as temperature and velocity of oxides based ethylene glycol/water-nanofluid are affected by nanoparticle volume fraction (χ), magnetic (M), and Casson parameters. Therefore, this study is intended to contribute a scientific admittance to the fluid mechanic's field. The heat transfers properties of the Newtonian fluids have been raised and enhanced by adding the nanoparticles (oxides) to them, and consequently the non-Newtonian fluids have been formed. In this article, only a free boundary layer in micropolar fluid and Casson nanofluid on stretching sheet is studied. Consequently, many sides can be proposed for future development, such as considering the other types of fluids flow likes Eyring Powell, viscoelastic, Jeffrey, and Maxwell fluids. In addition, this problem can be investigated over other bodies such as an elliptic circular cylinder. Moreover, the influences of physical heat transfer properties can be studied.

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