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SIMILARITY SOLUTION OF HEAT TRANSFER FOR THE UPPER-CONVECTED MAXWELL CASSON FLUID OVER A STRETCHING/SHRINKING SHEET WITH THERMAL RADIATION

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Abstract

The heat transfer for the upper-convected Maxwell Casson fluid over a stretching/shrinking sheet with thermal radiation is studied. With the help of similarity transformations, the governing equations are converted into nonlinear system of ordinary differential equations. Runge Kutta Fehlberg fourth-fifth order (RKF45) method is employed

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to obtain the numerical solutions for the skin friction coefficient, heat transfer coefficient, and velocity and temperature. The effects of various parameters such as the Casson parameter, Maxwell parameter, the radiation parameter, the suction parameter and the stretching and shrinking parameter are considered as distributions. Closed form analytic solutions are also obtained. The results obtained under the special cases are illustrated graphically and compared with previous results to support their validity.

1. Introduction

The study of heat transfer of non-Newtonian flows over a stretching sheet has various applications in engineering and industry, especially in extraction of crude oil from petroleum products. Also, it has the applications in the field of industrialization of plastic production, cooling of polymer chemistry, fiber technology, and elastic sheets. The heat transfer and flow properties of the Casson fluids in the presence of heat transfer are widely used in the processing of chocolate, foams, syrups, nail, toffee and many other foodstuffs (Ramachandra et al. [1]). Casson [2], in his pioneering work, introduced this model to simulate industrial inks. Later on, a substantial study has been done on the Casson fluid flow because of its important engineering applications. Mustafa et al. [3] have studied the heat transfer flow of a Casson fluid over an impulsive motion of the plate using the homotopy method. The exact solution of forced convection boundary layer Casson fluid flow toward a linearly stretching surface with transpiration effects is reported by Mukhopadhyay et al. [4]. In the same year, Subba et al. [5] considered the velocity and thermal slip conditions on the laminar boundary layer heat transfer flow of a Casson fluid past a vertical plate. Mahdy and Ahmed [6] studied the effect of magnetohydrodynamic on a mixed convection boundary flow of an incompressible Casson fluid in the stagnation point of an impulsively rotating sphere. The convective boundary layer flow of Casson nanofluid from an isothermal sphere surface is presented by Nagendra et al. [7]. Mehmood et al. [8] investigated the micropolar Casson fluid on mixed convection flow induced by a stretching sheet. Shehzad et al. [9] discussed the viscous chemical reaction effects on the MHD flow of a Casson fluid over a porous stretching sheet. Recently, Khalid et al. [10] developed exact solutions for unsteady MHD free convection flow of a Casson fluid past an oscillating plate. Amongst the various investigations on Casson fluid, the reader is referred to some new attempts made in Qasim and Noreen [11], Hussanan et al. [12] and Haq et al. [13] and the references therein.

On the other hand, the models of upper-convected Maxwell (UCM) fluid describes the behaviors of the viscoelastic fluids, which takes into account the stress relaxation that exists in the flow. The boundary layer flow and heat transfer due to stretching or shrinking sheet for the upper-convected Maxwell fluid have been considered and extended by many researches (see Sadeghy et al. [14], Sadeghy et al. [15], Hayat et al. [16], Alizadeh-Pahlavan et al. [17], Hayat et al. [18], Raftari and Yildirim [19], Mamaloukas et al. [20], Abel et al. [21], Mushtaq et al. [22], Lok et al. [23] and Jafar et al. [24]) in various ways.

The aim of this paper is to study the similarity solution of heat transfer for the upper-convected Maxwell Casson fluid over a stretching/shrinking sheet with thermal radiation. The sheet is subjected to variable magnetic field. Using some suitable similarity transformations, the governing nonlinear partial differential equations are reduced into nonlinear ordinary differential equations. Numerical solutions are obtained using Runge Kutta Fehlberg fourth-fifth order (RKF45) method. The results obtained under special cases are then compared with those in the literatures to support the validity of our result.

2. The Mathematics Formulation

Consider a steady two-dimensional boundary layer flow of an upper-convected Maxwell Casson fluid past a stretching/shrinking sheet at $y = 0$. Here, the heat transfer analysis is carried out in the presence of thermal radiation effects. The x-axis is taken along the stretching sheet and the y-axis is normal to the sheet. The flow is generated due to stretching of the sheet caused by the simultaneous application of two equal and opposite forces along the x-axis. It is also assumed that the rheological equation of state for an isotropic and incompressible flow of a Casson fluid can be written as (Hussanan et al. [25] and Alkasasbeh [26])

$$\tau_{ij} = \begin{cases} 2(\mu_B + p_y\sqrt{2\pi})e_{ij}, & \pi > \pi_c, \\ 2(\mu_B + p_y\sqrt{2\pi_c})e_{ij}, & \pi < \pi_c, \end{cases}$$

where $\pi = e_{ij}e_{ij}$, e_{ij} is the (i, j)th component of the deformation rate, μ_B is the plastic dynamic viscosity of the non-Newtonian fluid, π_c is a critical value of this product based on the non-Newtonian model and p_y is the yield stress of the fluid. Therefore, under these assumptions, the governing equations for the upper-convected Maxwell Casson fluid can be written as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \lambda\left(u^2\frac{\partial^2 u}{\partial x^2} + v^2\frac{\partial^2 u}{\partial y^2} + 2uv\frac{\partial^2 u}{\partial x \partial y}\right) = \nu\left(1 + \frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \tag{3}$$

subject to the boundary conditions:

$$u = U_w(x) = \varepsilon ax,\; v = v_w(x),\; T = T_w(x) = T_\infty + bx^2 \text{ at } y = 0,$$

$$u \to 0,\; T \to T_\infty \text{ as } y \to \infty \tag{4}$$

where a is a constant rate of stretching/shrinking, and u and v are the velocity components along the x-axis and y-axis, respectively. Meanwhile, T is the temperature of the fluid, T_∞ is the temperature of the ambient, ρ is the fluid density, σ is the electrical conductivity of the fluid, λ is the relaxation time parameter of the fluid, while ν is the kinematic viscosity of the fluid, C_p is the specific heat at a constant pressure, k is the thermal conductivity of the fluid, $v = v_w(x)$ is the wall mass suction velocity, $u = U_w(x) = \varepsilon ax$ is the stretching/shrinking velocity of the sheet where $\varepsilon = -1, 1$ is, respectively, for stretching and shrinking sheet and $\beta = \mu_B \sqrt{2\pi_c}/p_y$ is the parameter of the Casson fluid. Employing Rosseland approximation for radiation, the radiative heat flux is simplified as (Bataller [27])

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial \bar{y}}, \tag{5}$$

where σ^* and k^* are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively. We assume the temperature differences within the flow through the micropolar fluid such that the term T^4 may be expressed as a linear function of temperature. Hence, expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, we get

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4. \tag{6}$$

Invoking equations (3), (5) and (6), one can write

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\left(k+\frac{16\sigma^* T_\infty^3}{3k^*}\right)\frac{\partial^2 T}{\partial y^2}. \tag{7}$$

Introducing the following transformations:

$$\eta=\sqrt{(a/v)}\,y,\quad \psi=\sqrt{(av)}\,xf(\eta),\quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \tag{8}$$

where ψ is the stream function defined as $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$ which identically satisfies (1). Thus, we have

$$u=axf'(\eta),\quad v=\sqrt{\nu a}\,f(\eta), \tag{9}$$

so that the continuity equation (1) is automatically satisfied. Equations (2) and (7) can then be written as

$$\left(1+\frac{1}{\beta}\right)f'''+ff''-f'^2+\delta(2ff'f''-f^2f''')=0, \tag{10}$$

$$\frac{1}{Pr}\left(1+\frac{4}{3}R\right)\theta''+f\theta'-2f'\theta=0 \tag{11}$$

and the boundary conditions (4) become

$$f(0)=S,\ f'(0)=\varepsilon,\ \theta(0)=1 \text{ at } \eta=0,$$

$$f'(\eta)\to 0,\ \theta(\eta)\to 0 \text{ as } \eta\to\infty, \tag{12}$$

where primes denote differentiation with respect to η. The dimensionless Maxwell parameter δ, Prandtl number Pr, suction parameter S and radiation parameter R are given by

$$\delta=\lambda a,\quad Pr=\frac{\mu c_\rho}{k},\quad S=-\frac{v_w(x)}{(a\nu)^{1/2}},\quad R=\frac{\alpha k^*\rho c_\rho}{4\sigma^* T_\infty^3}. \tag{13}$$

3. Results and Discussion

The numerical solutions of the nonlinear ordinary differential equations (10) and (11) along with boundary conditions (12) are solved using Runge-Kutta Fehlberg (RKF45) method in Maple software because this scheme is stable, easy to implement and self-starting. In order to verify the accuracy of the present method, the results are compared with those reported by Sadeghy et al. [15] as shown in Table 1. It has been found that they are in good agreement.

Table 1. The comparison of the skin friction coefficient $f''(0)$ for different values of Maxwell parameter δ when $\beta \to \infty$, $S = 0$, $\varepsilon = 1$, $R = 0$ and $Pr = 1$

	$f''(0)$	
δ	Sadeghy et al. [15]	Present results
0	−1.0000	−1.000072
0.2	−1.0549	−1.054890
0.4	−1.10084	−1.100051
0.6	−1.00150	−1.001549
0.8	−1.19872	−1.198661

Table 2. Values of the skin friction coefficient $f''(0)$ and heat transfer coefficient $-\theta'(0)$ for different values of Casson parameter β, suction parameter S, Maxwell parameter δ and stretching sheet ε when $R = 1$ and $Pr = 1$

β	S	δ	ε	$f''(0)$	$-\theta'(0)$
0.1	0.5	0.05	1	−0.320839	1.088692
0.5				−0.625815	1.017608
1				−0.783190	0.984162
2				−0.922065	0.957342
1	0.5	0.05	1	−0.556646	0.790951
	0			−0.660222	0.881365

	0.5			−0.783190	0.984162
	1			−0.925438	1.100248
1	0.5	0.05	1	−0.783190	0.984162
		0.1		−0.730487	1.003662
		0.2		−0.636582	1.032765
		0.3		−0.551690	1.055231
1	0.5	0.05	0.5	−0.252759	0.755208
			1	−0.783190	0.984162
			2	−2.211916	1.325199
			3	−4.017583	1.592335

Table 2 presents the values of the skin friction coefficient $f''(0)$ and heat transfer coefficient $-\theta'(0)$ for different values of Casson parameter β, suction parameter *S*, Maxwell parameter δ and stretching sheet ε when $R = 1$ and $Pr = 1$. It can be seen that when the suction parameter *S* and stretching sheet ε increase, the skin friction coefficient $f''(0)$ decreases and heat transfer $-\theta'(0)$ increases. Moreover, from this table, note that when Maxwell parameter δ increases, the heat transfer $-\theta'(0)$ and skin friction coefficient $f''(0)$ increase. In addition, when the Casson parameter β increases, it leads to a decrease in the heat transfer $-\theta'(0)$ and skin friction coefficient $f''(0)$.

The velocity and temperature profiles for different values of the Casson parameter β are shown in Figures 1 and 2. Figure 1 indicates that an increase in β tends to decrease in the velocity $f'(\eta)$. It is true because β appeared in the shear term of the momentum equation (10), and an increase in β implies a decrease in yield stress of the Casson fluid. Physically, an increase in Casson parameter means a decrease in yield stress and increases the plastic dynamic viscosity of the fluid, which makes the momentum boundary layer thicker. This effectively slows down the fluid motion. It is noticed from Figure 2 that the temperature $\theta(\eta)$ increases as the values of β increase.

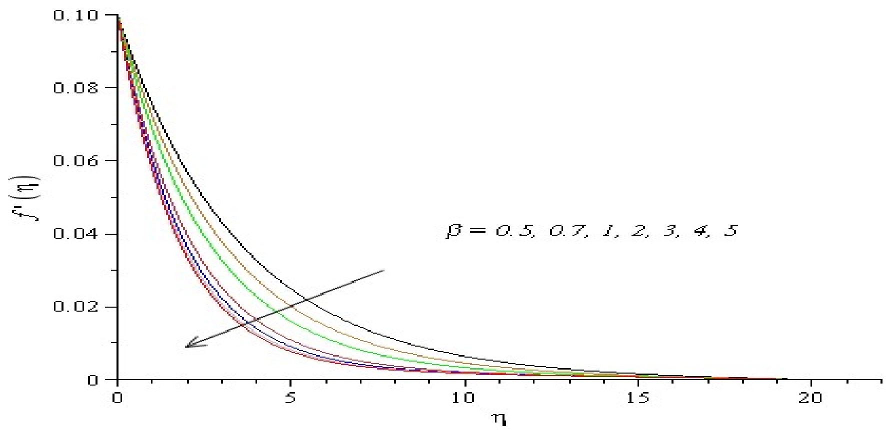

Figure 1. The effect of Casson parameter β on the velocity profiles $f'(\eta)$ for $\varepsilon = 0.1$, $S = 0.5$, $Pr = 1$, $R = 1$ and $\delta << 1$.

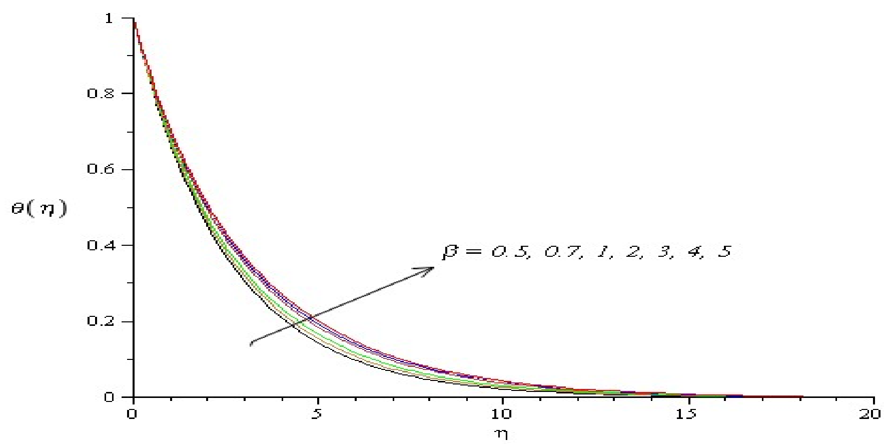

Figure 2. The effect of Casson parameter β on the temperature profiles $\theta(\eta)$ for $\varepsilon = 0.1$, $S = 0.5$, $Pr = 1$, $R = 1$ and $\delta << 1$.

Figure 3 illustrates the effect of radiation parameter R on the temperature profile $\theta(\eta)$ for small value of Maxwell parameter δ. It is found that as the value of radiation parameter increases, the temperature profile increases. This agrees to the physical properties of the radiation which is clarified in equation (7).

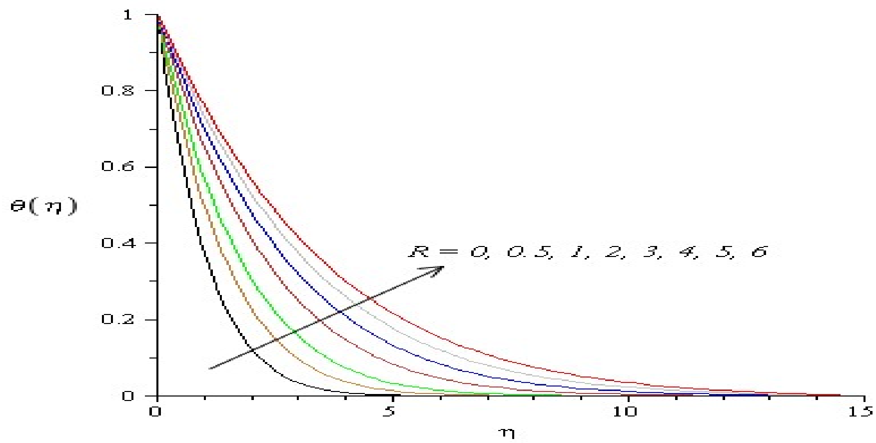

Figure 3. The effect of radiation parameter R on the temperature profiles $\theta(\eta)$ for $\varepsilon = 0.1$, $S = 0.5$, $Pr = 1$, $R = 1$ and $\delta = 0.05$.

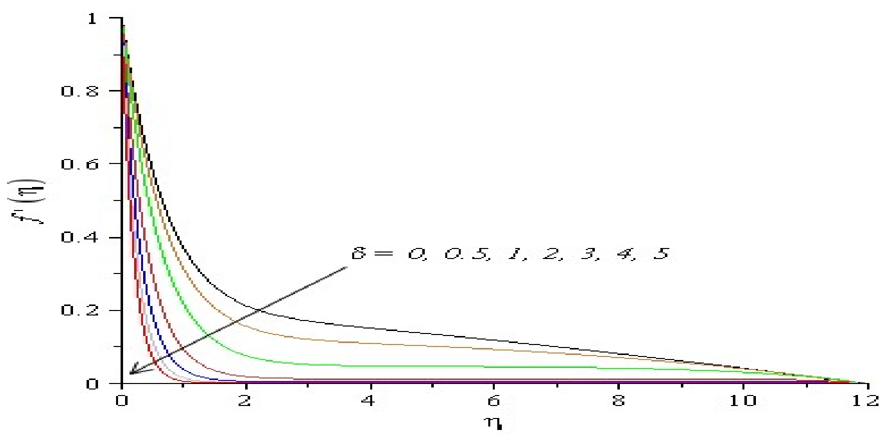

Figure 4. The effect of Maxwell parameter δ on the velocity profiles $f'(\eta)$ for $\varepsilon = 1$, $S = 0.5$, $Pr = 1$, $R = 1$ and $\beta = 1$.

The effect of Maxwell parameter δ on the velocity profiles $f'(\eta)$ and the temperature profiles $\theta(\eta)$ are shown in Figures 4 and 5. It is found that an increasing of Maxwell parameter δ leads to decreasing on velocity, temperature profile and the thermal boundary layer thickness.

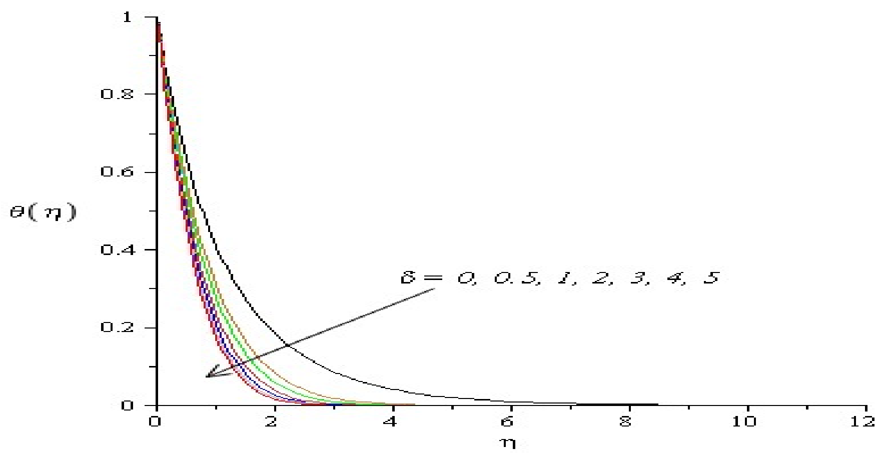

Figure 5. The effect of Maxwell parameter δ on the temperature profiles $\theta(\eta)$ for $\varepsilon = 1$, $S = 0.5$, $Pr = 1$, $R = 1$ and $\beta = 1$.

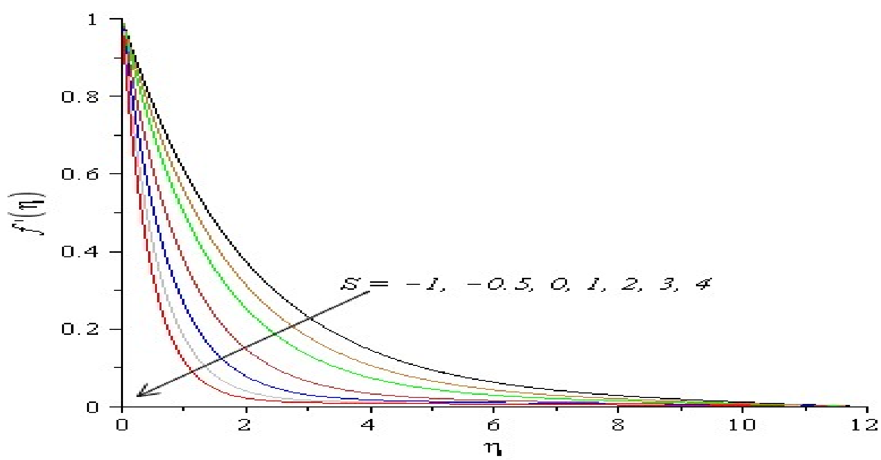

Figure 6. The effect of suction parameter S on the velocity profiles $f'(\eta)$ for $\varepsilon = 1$, $\delta = 0.005$, $Pr = 1$, $R = 1$ and $\beta = 1$.

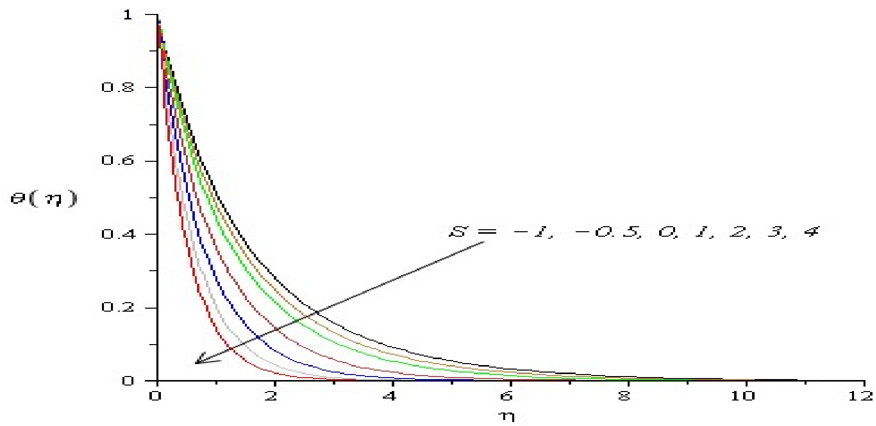

Figure 7. The effect of suction parameter S on the temperature profiles $\theta(\eta)$ for $\varepsilon = 1$, $\delta = 0.005$, $Pr = 1$, $R = 1$ and $\beta = 1$.

Figures 6 and 7 show the effect of the suction parameter S on the velocity and temperature profiles. An increase in suction parameter S leads to decrease in velocity and temperature profiles.

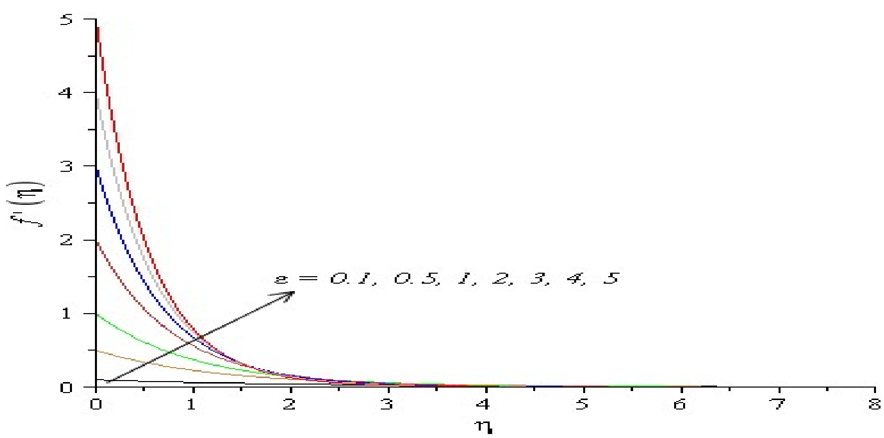

Figure 8. The effect of stretching sheet ε on the velocity profiles $f'(\eta)$ for $S = 1$, $\delta = 0.005$, $Pr = 1$, $R = 1$ and $\beta = 1$.

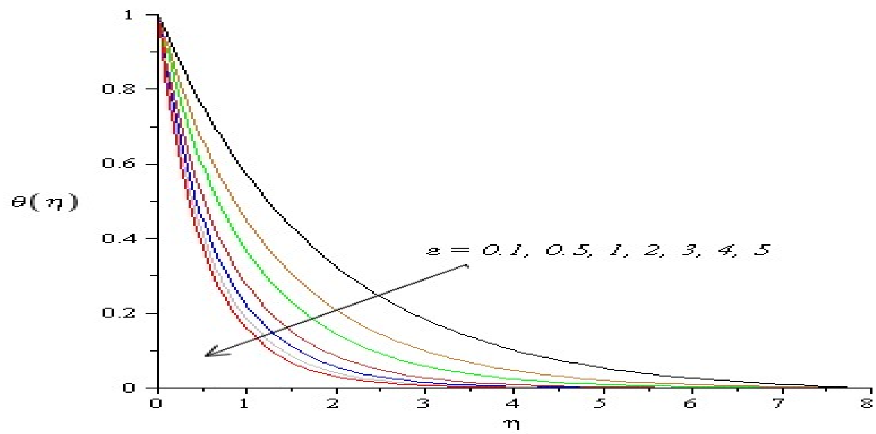

Figure 9. The effect of stretching sheet ε on the temperature profiles $\theta(\eta)$ for $S = 1$, $\delta = 0.005$, $Pr = 1$, $R = 1$ and $\beta = 1$.

Figures 8 and 9 present the effect of the stretching sheet ε on velocity and temperature profiles. It is observed that the velocity increases with increase in stretching sheet ε and decrease in temperature profile. Physically, for fixed values of ε corresponding to the stretching of the surface, an increase in *a* implies an increase in straining motion near the stagnation point resulting in increased acceleration of the external stream and this leads to thinning of the boundary layer by increasing ε.

4. Conclusion

Numerical solution of the coupled nonlinear ordinary differential equations derived from similarity transformation for the problem of stagnation-point flow and heat transfer for the upper-convected Maxwell Casson fluid over a stretching/shrinking sheet with thermal radiation is investigated using Runge Kutta Fehlberg fourth-fifth order (RKF45) method. The numerical computation is carried out for various values of non-dimensional physical parameters, namely, Casson parameter β, radiation parameter R, suction parameter S, Maxwell parameter δ and stretching sheet ε. The main findings are summarized as follows:

- An increase in the Casson parameter leads to decrease in the skin friction coefficient, heat transfer coefficient and velocity profiles and increase in the temperature profiles. Moreover, when radiation parameter R increases, the temperature profiles increase.
- An increase in the Maxwell parameter leads to increase in the skin friction coefficient and heat transfer coefficient and decrease in the velocity and temperature profiles.
- An increase in the suction parameter leads to decrease in the skin friction coefficient, velocity and temperature profiles and increase in the local heat transfer coefficient.
- An increase in the value stretching sheet leads to decrease in the skin friction coefficient, local heat transfer coefficient and temperature profiles and decrease in the velocity profiles.

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