Significant Modification of Pairwise−ω −continuous Functions with Associated Concepts

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Abstract: - The continuity is generalized by the notion of ω −continuous functions. In this research, we present a new weaker form for continuous functions called pairwise ω −continuous functions. Additionally, we define pairwise barely ω −continuous functions, a new, weaker form of barely ω −continuous functions. We study the basic characteristics and impacts of pairwise ω −continuous functions, clarifying their connection with typical continuity and providing perspectives on the wider field of topological analysis. It explores related ideas like the ω −limit, which describes how sequences behave over certain conditions when the function is applied. In addition, the concepts highlight the importance of pairwise ω −continuous functions in theoretical and practical conditions by discussing their relationships with other functional structures. An extensive number of demonstrative examples will be presented, along with the new results and theorems about pairwise barely ω −continuous and pairwise ω −continuous functions that generalize.

Key-Words: - Bitopological spaces, pair−ω −closed, pair−lindelöf, pair−ω −contiuous, pair−almost contenuous functions, pair−weakly continuous functions, pair−sequentially continuous functions, pair−barely continuous functions, pair−barely ω −continuous functions.

Received: May 31, 2019. Revised: May 4, 2020. Accepted: May 22, 2020. Published: May 29, 2020

1 Introduction

Pairwise-ω-continuous function analysis is seen as an extension of a similar topic in topological spaces, whose was a development of topological spaces with simply one topology.

The primary goal for including ω-continuous functions in the topology is to generalize certain characteristics of continuous functions. In generic topology, compactness and lindelöf are the most basic components. Moreover, these two concepts have a great deal of utility in implementations; topology is employed in various fields of analysis as well as logical mathematics, and continuous function conceptions are utilized widely in mathematical analysis. Since in, [1], first proposed the idea of a bitopological space in 1963, certain topological properties of single topology such as compactness and paracompactness have been generalised to bitopological spaces, as well as separation axioms, connectedness, function types, and more concepts. In 1982, [2],demonstrated that the lindelöf property is maintained by counter images of ω −closed mappings using lindelöf counter images for points. He introduced a new type of mapping, called ω −closed mappings, that are strictly weaker than closed mappings. Also, he demonstrated that if the inverse image of every point in the range is paracompact with respect to the domain (lindelöf, respectively), then the paracompactness (strongly paracompactness) property is maintained by obtaining counter images of ω −closed mappings using regular domains. Moreover, he defined P^* −space as a generalisation of P −space and demonstrated that for any P^* −space Y, the projection of $P: E \times V \rightarrow V$ is ω −closed if and only if X is in a lindelöf space. Additionally, he derived various product theorems related to lindelöf for paracompact and strongly paracompact spaces using P^* −spaces. He then discusses a few different examples that relate to the definitions and theorems that are provided. In 2021, in, [3], introduces pairwise ω −perfect functions as well as pairwise $M - \omega$ −perfect functions, describing their features and searching for homeomorphisms between various bitopological spaces under their influence. Finally, he provides the product theorem characterizations. In 2022, [4], presented the idea of weakly ω −continuous functions within bitopological spaces as a generalisation of $u - \omega$ −continuous functions, and they obtained numerous features and some of their characteristics. Also, in the same year 2022, [5], proposed the idea of nearly ω −continuous functions within bitopological spaces as a generalization of $u - \omega$ −continuous functions, and they established several results and some of their characteristics. While, in 2022, [6], created a new type of function via ω −open sets that he named rarely ω −continuous function and examined many characteristics of this function. Throughout this research, we will use pair−compact, which refers to pairwise compact and pair− to denotes pairwise. If (E, α_1, α_2)represents a bitopological space and $A \subseteq E$; then the closure for A with regard to α_1 and α_2, respectively, will be indicated by the symbols $cl_1(A)$ and $cl_2(A)$. Let A be a subset of E and let (E, α) be any topological space, a point $e \in (E, \alpha_1, \alpha_2)$is known as the condensation point of A if $H \cap A$ is uncountable set, for any $H \in \alpha$ with $e \in H$. The following describes the way, [7], described ω −closed sets as well as ω −open sets. If A has all its condensation points, it is said to as ω −closed. ω −open is the complement of a ω −closed set. Additionally, the intersection with all ω −closed sets that contain A will be indicated by $cl^{\omega}(A)$. Also, the space (E, α_1, α_2), or just E refers to any bitopological space on which, unless otherwise specified, no separation axioms can be taken for assumed in this resaerch. Pair−ω −continuous functions are a new kind of weakened continuous function that is introduced in this study. Additionally, we define pair−barely ω −continuous functions, a new, weakened type of barely continuous functions. First, in Section 3, we establish numerous characteristics of pair−ω −continuous functions. Several examples are provided in Section 4 to show how pair−ω −continuous functions and certain weakened forms of pairwise continuous functions are connected. In the end, we show whether a pair−ω-continuous image that represents a pair−lindelöf space is also a pair−lindelöf space in Section 5 by presenting a new, basic characterization of pair−lindelöf spaces. Furthermore, as demonstrated in Section 5, a pairwise only barely ω −continuous image is hereditarily. Applications of continuous functions with different topological spaces can be found in [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24], [25], [26], [27], [28], [29].

2 Preliminaries

This section presents some significant concepts along with specifics are covered that used within the research.

Definition 2.1, [2] A function $\psi: (E, \alpha_1, \alpha_2) \rightarrow (R, \beta_1, \beta_2)$ is said to be pair−continuous if the functions $\psi_1: (E, \alpha_1) \rightarrow (R, \beta_1)$ and $\psi_2: (E, \alpha_2) \rightarrow (R, \beta_2)$ are continuous.

Definition 2.2, [2] A function $\psi: (E, \alpha_1, \alpha_2) \rightarrow (R, \beta_1, \beta_2)$ is said to be pair−closed if the functions $\psi_1: (E, \alpha_1) \rightarrow (R, \beta_1)$ and $\psi_2: (E, \alpha_2) \rightarrow (R, \beta_2)$ are closed.

Definition 2.3, [2] Let (E, α_1, α_2) be any bitopological space. Then, a cover h of the space (E, α_1, α_2) is called $\alpha_1\alpha_2$ −open if $h \subset \alpha_1 \cup \alpha_2$. Moreover, h is called pair−open if h has at least one-nonempty member of α_2.

Definition 2.4, [2] Let (E, α_1, α_2) be any bitopological space. Then, the space is called pair$-$lindelöf if any pair$-$open cover of (E, α_1, α_2) has a countable subcover.

Definition 2.5, [2] Let (E, α_1, α_2) be any bitopological space. Then, the space is called $s-$lindelöf if any $\alpha_1\alpha_2-$open cover of (E, α_1, α_2) has a countable subcover.

Definition 2.6, [2] Let (E, α_1, α_2) be any bitopological space. Then, the space is called pair$-$countably compact if any countable pair$-$open cover of (E, α_1, α_2) has a finite subcover.

Definition 2.7, [2] Let (E, α_1, α_2) be any bitopological space. Then, the space is called semi$-$countably compact if any countably $\alpha_1\alpha_2-$open cover of (E, α_1, α_2) has a finite subcover.

Definition 2.8, [2] A function $\psi: (E, \alpha_1, \alpha_2) \to (R, \beta_1, \beta_2)$ is said to be pair$-$lindelöf if for any pair$-$lindelöf closed subset H of the space (R, β_1, β_2), $\psi^{-1}(H)$ is pair$-$lindelöf.

Definition 2.9, [10] A function $\psi: (E, \alpha_1, \alpha_2) \to (R, \beta_1, \beta_2)$ is said to be semi$-$lindelöf if for any semi$-$lindelöf closed subset H of the space (R, β_1, β_2), $\psi^{-1}(H)$ is semi$-$lindelöf.

Definition 2.10, [10] Let (E, α_1, α_2) be any bitopological space, then we called α_1 represents locally lindelöf with a respect to α_2, if there exists α_1 nbd h_y of x for any $y \in (E, \alpha_1, \alpha_2)$, such that $\bar{h}_y^{\alpha_2}$ is pair$-$lindelöf.

Definition 2.11, [10] Let (E, α_1, α_2) be any bitopological space, then the space called pair$-$locally lindelöf if and only if α_n is locally lindelöf with respect to α_m, where $n, m = 1,2$ and $n \neq m$.

Definition 2.12, [22] A function $\psi: (E, \alpha_1, \alpha_2) \to (R, \beta_1, \beta_2)$ is said to be pair$-\omega-$closed if the functions represent pair$-$closed sets onto pair$-\omega-$closed sets.

Definition 2.13, [24] A function $\psi: (E, \alpha_1, \alpha_2) \to (R, \beta_1, \beta_2)$ is called pair$-$weakly continuous if for any pair$-$open set $A \subset R$, there is $\psi^{-1}(A)$ is pair$-\omega-$open.

Definition 2.14, [25] A function $\psi: (E, \alpha_1, \alpha_2) \to (R, \beta_1, \beta_2)$ is called pair$-$almost continuous if for any $y \in E$ and any β_i-open set v of R containing $\psi(y)$ there exists α_i-open set u containing y such that $\psi(u) \subset \beta_i - \text{int}(\beta_i - Cl(v))$.

Definition 2.15, [25] A function $\psi: (E, \alpha_1, \alpha_2) \to (R, \beta_1, \beta_2)$ is called pair$-$weakly continuous if for any $y \in E$ and any β_i-open set v of R containing $\psi(y)$ there exists α_i-open set u containing y such that $\psi(u) \subset \beta_i - v$.

Recall that if an arbitrary neighbourhood (in simply nbd) for a point y in a space E includes an uncountable subset of the set $A \subset E$, then the point y is known as a condensation point of a set A.

Definition 2.16, [29] Let (E, α_1, α_2) be any bitopological space and $A \subseteq E$. The space is said to be pair$-\omega-$open if for any $a \in A$ there is a pair$-$open subset of h_a containing a such that the set $h_a - A$ is countable. Moreover, the complement set of pair$-\omega-$open is called pair$-\omega-$closed set. The family of all pair$-\omega-$open (respectively pair$-\omega-$closed) subsets of (E, α_1, α_2) is represented as pair$-\omega - BO(E)$,(respectively pair$-\omega - BC(E)$). Additionally, the family of all pair$-\omega-$open of the space (E, α_1, α_2) containing a is represented as pair$-\omega - BO(E; a)$.

Definition 2.17, [29] Let (E, α_1, α_2) be any bitopological space and $A \subseteq E$. The space is said to be semi$-\omega-$open if for any $a \in A$ there exists $\alpha_1\alpha_2-$open subset of h_a containing a such that the set $h_a - A$ is countable. Moreover, the complement set of semi$-\omega-$open is called semi$-\omega-$closed set. The family of all semi$-\omega-$open (respectively semi$-\omega-$closed) subsets of (E, α_1, α_2) is represented as semi$-\omega - BO(E)$ (respectively semi$-\omega - BC(E)$). Additionally, the family of all semi$-\omega-$open of the space (E, α_1, α_2) containing a is represented as semi$-\omega - BO(E; a)$.

Definition 2.18, [29] A function $\psi: (E, \alpha_1, \alpha_2) \to (R, \beta_1, \beta_2)$ is said to be semi$-\omega-$closed if the functions represent semi$-$closed sets onto semi$-\omega-$closed sets.

Theorem 2.19, [29] (i) Every pair$-\omega-$subset of the pair$-$lindelöf space is pair$-$lindelöf.
(ii) Let function $\psi: (E, \alpha_1, \alpha_2) \to (R, \beta_1, \beta_2)$ be any pair$-$function. Then the following facts are equivalent: (a) ψ is pair$-\omega-$closed; (b) for any $r \in (R, \beta_1, \beta_2)$ and each pair$-$open set H, where $\psi^{-1}(r) \subset H$, there exists pair$-\omega-$open set H_r such that $r \in H_r$ and $\psi^{-1}(H_r) \subset H$.

(iii) Every pair−lindelöf, pair$-\omega-$open subset B of (E,α_1,α_2) represent $G\setminus A$, which G is pair−open and A is countable set such that B is G_δ −set.

3 Properties of Pairwise ω-Continuous Functions

Additional results regarding the topological attributes of pairwise-ω-continuous functions are presented in this section. Following this, we will talk about the idea of pairwise-ω-continuous functions, extract some of their traits, and explain how they relate to various kinds of pairwise-ω-continuous functions, providing examples to demonstrate for every instance. There will also be discussion and proof of other theories on this topic.

Definition 3.1 A function $\psi:(E,\alpha_1,\alpha_2)\rightarrow(R,\beta_1,\beta_2)$ is said to be pair$-\omega-$continuous at the point $a\in(E,\alpha_1,\alpha_2)$, if for any pair-open set B containing $\psi(a)$, there is pair$-\omega-$open set A containing a such that $\psi(A)\subset B$. Moreover, a function ψ is called pair$-\omega-$continuous on (E,α_1,α_2), if the function ψ is pair$-\omega-$continuous at any point of (E,α_1,α_2).

The subsequent pair of instances demonstrates that pairwise-ω-continuous functions at a specific point are not always pairwise almost continuous at the identical place, and pairwise almost continuous functions at a particular point are not always pairwise ω-continuous at the identical location.

Example 3.2
Let the set $\mathbb{R}$ represents the real numbers and let α_u is the usual topology on $\mathbb{R}$. Providing the function $\psi:(\mathbb{R},\alpha_u,\alpha_u)\rightarrow(\mathbb{R},\alpha_u,\alpha_u)$, which is defined as ψ (z)=0, in the case of a z is rational, and ψ (z)=1, in the case of a z is irrational. Therefore at each irrational number, ψ is pairwise-ω-continuous; at every single real number, nevertheless, ψ is not pairwise almost continuous.

Example 3.3
Let the set $\mathbb{R}$ represents the real numbers and let α_u is the usual topology on $\mathbb{R}$. Consider $Z=\{n,m\}$, with the topology $\alpha=\{Z,\varphi,\{n\}$. Consider $\psi:(\mathbb{R},\alpha_u,\alpha_u)\rightarrow(\mathbb{R},\alpha,\alpha)$, which is defined as ψ (z)=a, in the case of a z is rational, and ψ (z)=b, in the case of a z is irrational. nevertheless, ψ is pairwise almost continuous at every rational number, nevertheless, ψ is not pairwise-ω-continuous.

Any pair−continuous function is obviously pair$-\omega-$continuous. The example that follows, nevertheless, demonstrates that this need not be necessary.

Example 3.4 Let α be any topology on $\mathbb{R}$ where n neighborhoods of every nonzero point are as in the form of usual topology, whereas n neighborhoods of H will represent as A/K, with A is a neighborhoods of H in the usual topology and $K=\left\{\frac{1}{m},m=1,2,...\right\}$. Now, let α_u be a usual topology on $\mathbb{R}$. Let the identity function be denoted by $\psi:(\mathbb{R},\alpha_u,\alpha_u)\rightarrow(\mathbb{R},\alpha,\alpha)$. Then, while ψ is clearly pair−continuous, it cannot be pair$-\omega-$continuous at H as a result.

Note that if (E,α,α) represents a bitopological space, then the family of every one of ω −open sets represent a topology α_ω, which is finer than α. Therefore, $\psi:(E,\alpha,\alpha)\rightarrow(R,\beta_1,\beta_2)$ is pair-ω-continuous if and only if $\psi:(E,\alpha_\omega,\alpha_\omega)\rightarrow(R,\beta_1,\beta_2)$ is pair−continuous. The next theorems are then stated with simplicity.

Theorem 3.5 Let $\psi:(E,\alpha_1,\alpha_2)\rightarrow(R,\beta_1,\beta_2)$ be a function. Then we have the equivalent statements as follows:
(1) the function ψ is pair$-\omega-$continuous,
(2) the function $\psi:(E,\alpha_\omega,\alpha_\omega)\rightarrow(R,\beta_1,\beta_2)$ is pair−continuous,
(3) for any pair−open set A of (R,β_1,β_2), $\psi^{-1}(A)$ is pair$-\omega-$open set in (E,α,α).
(4) for any pair−closed set B of (R,β_1,β_2), $\psi^{-1}(B)$ is pair$-\omega-$closed set in (E,α,α).

Theorem 3.6 If $\psi_1:(E,\alpha_1,\alpha_2)\rightarrow(R,\beta_1,\beta_2)$ is pair$-\omega-$continuous, and $\psi_2:(R,\beta_1,\beta_2)\rightarrow(S,\gamma_1,\gamma_2)$ is pair−continuous, then $\psi_1\circ\psi_2$ is pair$-\omega-$continuous.

Theorem 3.7 If $\psi_1:(E,\alpha_1,\alpha_2)\rightarrow(R,\beta_1,\beta_2)$ is pair$-\omega-$continuous and $E_1\subset E$, then the function $\psi/_{E_1}$ is pair$-\omega-$continuous.

Theorem 3.8 If $\psi_1:(E,\alpha_1,\alpha_2)\rightarrow(R,\beta_1,\beta_2)$ is pair−continuous and $\psi_2:(E,\alpha_1,\alpha_2)\rightarrow(S,\gamma_1,\gamma_2)$ be pair$-\omega-$continuous, then $\psi_{1,2}:(E,\alpha_1,\alpha_2)\rightarrow(R\times S,\beta_1\times\gamma_1,\beta_2\times\gamma_2)$ represents as $\psi_{1,2}(y)=\left(\psi_1(y),\psi_2(y)\right)$ is pair$-\omega-$continuous function.

Proof Let $u_1\times v_1$ be basics of $\beta_1\times\gamma_1$ −open subset of $(R\times S,\beta_1\times\gamma_1,\beta_2\times\gamma_2)$. Then $\psi_{1,2}^{-1}(u_1\times v_1)=\psi_1^{-1}(u_1)\cap\psi_2^{-1}(v_1)$. Now, since

$\psi_1: (E, \alpha_1, \alpha_2) \to (R, \beta_1, \beta_2)$ be pair−continuous and $\psi_2: (E, \alpha_1, \alpha_2) \to (S, \gamma_1, \gamma_2)$ be pair$-\omega$ −continuous functions, $\psi_1^{-1}(u_1)$ is β_1 −open and $\psi_2^{-1}(v_1)$ is $\gamma_1 - \omega$ −open. Thus, $\psi_{1,2}^{-1}(u_1 \times v_1)$ is $\alpha_1 - \omega$ −open. In the same way for $u_2 \times v_2$ a basics $\beta_1 \times \gamma_1$ −open subset of $(R \times S, \beta_1 \times \gamma_1, \beta_2 \times \gamma_2)$. Hence, the function $\psi_{1,2}$ is pair$-\omega$ −continuous.

Theorem 3.9 Let the function $\psi_{1,2}: (E, \alpha_1, \alpha_2) \to (E_1 \times E_2, \alpha_1 \times \alpha_2)$ is defined as $\psi_{1,2}(y) = (\psi_1(y), \psi_2(y))$, where $\psi_{1,2}: (E, \alpha_1, \alpha_2) \to (E_{1,2}, \alpha_{1,2})$. Then $\psi_{1,2}$ is pair$-\omega$ −continuous if and only if ψ_1 and ψ_2 are two pair$-\omega$ −continuous function.

Proof ⇒) Assume that $\psi_{1,2}$ is pair$-\omega$ −continuous. As $\psi_1 = J_1 o \psi_{1,2}$, where the projection function $J_1: (E_1 \times E_2, \alpha_1 \times \alpha_2) \to (E_1, \alpha_1)$, so by theorem (3.4) we have ψ_1 is pair$-\omega$ −continuous. In the same way, as we can demonstrate that ψ_2 is pair$-\omega$ −continuous.
⇐) Assume that the functions ψ_1 and ψ_2 are two pair$-\omega$ −continuous. Let H be $\alpha_1 \times \alpha_2$ subset of $(E_1 \times E_2, \alpha_1 \times \alpha_2)$ such that $H = v \times E_2$, where the set v is a α_1 −open of (E_1, α_1). Thus $\psi_{1,2}^{-1}(H) = \psi_1^{-1}(v)$. But the function ψ_1 is pair$-\omega$ −continuous, so the set $\psi_1^{-1}(v)$ is pair$-\omega$ −open. Therefore, the function $\psi_{1,2}: (E, \alpha_1, \alpha_2) \to (E_1 \times E_2, \alpha_1 \times \alpha_2)$ is pair$-\omega$ −continuous.

Theorem 3.10 Let $\psi: (E, \alpha_1, \alpha_2) \to (R, \beta_1, B_2)$ be any function and $y \in (E, \alpha_1, \alpha_2)$. If there exist a pair−open set V of (E, α_1, α_2) such as $y \in V$, and the function $\psi/_V$ is pair$-\omega$ −continuous at y.

Proof Assume that the subset W is β_1 −open of (R, β_1, β_2) containing $\psi(y)$. Since the function $\psi/_V$ is pair$-\omega$ −continuous at y, then there is a $\alpha_1 - \omega$ −open set O of V containing y such that $\psi(O) = \left(\psi/_V\right)(O) \subset W$. Since the set V is pair−open of (E, α_1, α_2) and $y \in V$, so since If $\psi_1: (E, \alpha_1, \alpha_2) \to (R, \beta_1, \beta_2)$ is pair$-\omega$ −continuous and $E_1 \subset E$, then the function $\psi/_{E_1}$ is pair$-\omega$ −continuous , we have $O \in \alpha_\omega$ containing y. In the same way of the proof, for the subset Z is β_2-open of (R, β_1, β_2) containing $\psi(y)$. Therefore, the function ψ is pair$-\omega$ −continuous at y.

Corollary 3.11 Let the function $\psi: (E, \alpha_1, \alpha_2) \to (R, \beta_1, \beta_2)$ and $N = \{N_\epsilon : \epsilon \in \Upsilon\}$ be any pair−open cover of (E, α_1, α_2), such as $N_\epsilon \in \alpha_\omega$ for any $\epsilon \in \Upsilon$. If the function $\psi/_{N_\epsilon}$ is pair−continuous for any $\epsilon \in \Upsilon$, then ψ is pair$-\omega$ −continuous.

It can be observed in the following example that two pair$-\omega$ −open sets do not necessarily have to be pair$-\omega$ −open products.

Example 3.12 Let $E = (\mathbb{R}, \alpha_u, \alpha_u)$, $S = (\mathbb{R}, \alpha_u, \alpha_u)$ where the set $\mathbb{Q}$ represents the irrational numbers of E and $I = (0,1)$ is subset of S. Then $\mathbb{Q}$ is pair$-\omega$ −open set in E and I is pair−open in S, so it is pair$-\omega$ −open set. Moreover, $\mathbb{Q} \times I$ is not pair$-\omega$ −open set.

Question: Let the functions $\psi_1: (E_1, \alpha_1, \alpha_2) \to (R_1, \beta_1, \beta_2)$ and $\psi_2: (E_2, \alpha_1, \alpha_2) \to (R_2, \beta_1, \beta_2)$ be two pair$-\omega$ −continuous. Is the function $\psi_{1,2} = (\psi_1 \times \psi_2): (E_1 \times E_2, \alpha_1 \times \alpha_2, \alpha_1 \times \alpha_2) \to (R_1 \times R_2, \beta_1 \times \beta_2, \beta_1 \times \beta_2)$ pair$-\omega$ −continuous. In fact, the answer will be negative. The example below demonstrates this.

Example 3.13 Let the set $\mathbb{R}$ represents the real numbers and let α_u is the usual topology on $\mathbb{R}$. Assume that $E = \{c, d\}$ such that $\alpha = \{\emptyset, E, \{c\}\}$, and the function $\psi_1: (\mathbb{R}, \alpha_u, \alpha_u) \to (E, \alpha, \alpha)$ represents as $\psi_1(y) = c$ is irrational, $\psi_1(y) = d$ is rational. Now, let the identity function is $\psi_2: (\mathbb{R}, \alpha_u, \alpha_u) \to (\mathbb{R}, \alpha_u, \alpha_u)$. Thus, it clearly that the functions ψ_1 and ψ_2 are two pair$-\omega$ −continuous. Now, suppose that the function $\psi_{1,2} = (\psi_1 \times \psi_2): (\mathbb{R} \times \mathbb{R}, \alpha_u \times \alpha_u, \alpha_u \times \alpha_u) \to (E_1 \times \mathbb{R}, \alpha \times \alpha_u, \alpha \times \alpha_u)$ represents as $\psi_{1,2}(y_1, y_2) = (\psi_1(y_1), \psi_2(y_2))$. Let the set $U = \{c\} \times (0,1)$ is $\alpha \times \alpha_u$ −open in $E_1 \times \mathbb{R}$. So, $\psi_{1,2}^{-1}(U) = \psi_1^{-1}(\{c\}) \times \psi_2^{-1}((0,1)) = \mathbb{Q} \times (0,1)$, but by previous example (3.11), we have $\psi_{1,2}^{-1}(U)$ is not pair$-\omega$ −open. Therefore, the function $\psi_{1,2}$ is not pair$-\omega$ −continuous.

Definition 3.14 A space (E, α_1, α_2) is called pair$-\omega$ −compact, if any pair$-\omega$ −open cover of the space E has a finite subcover.

Definition 3.15 A space (E, α_1, α_2) is called pair$-\omega$ −lindelöf, if any pair$-\omega$ −open cover of the space E has a countable subcover.

Definition 3.16 A space (E, α_1, α_2) is called pair$-\omega-$closed compact, if any pair$-\omega-$closed cover of the space E has a finite subcover.

Definition 3.17 A space (E, α_1, α_2) is called a countably pair$-\omega-$closed compact, if any countable pair$-\omega-$closed cover of the space E has a finite subcover.

Theorem 3.18 Let the surjection function $\psi: (E, \alpha_1, \alpha_2) \to (R, \beta_1, \beta_2)$ is pair$-\omega-$continuous. Then the next statements are hold:
(1) If (R, β_1, β_2) is pair$-\omega-$lindelöf space, then (E, α_1, α_2) is pair$-$lindelöf,
(2) If (R, β_1, β_2) is countably pair$-\omega-$compact space, then (E, α_1, α_2) is pair$-$countably compact.

Proof (1) Let $\tilde{u} = \{u_\epsilon : \epsilon \in \Upsilon\}$ be any pair$-\omega-$open cover of (E, α_1, α_2), since ψ is surjection function and pair$-\omega-$continuous, then for any $r \in (R, \beta_1, \beta_2)$, there exists Υ_r, for any $\Upsilon_r \in \Upsilon$ is countable subcover such that $\psi^{-1}(r) \subseteq \bigcup_{\epsilon \in \Upsilon_r} \{v_\epsilon : \epsilon \in \Upsilon_r\} \cup \bigcup_{\epsilon \in \Upsilon_r^*} \{\omega_\epsilon : \epsilon \in \Upsilon_r^*\}$, where the set $\{v_\epsilon : \epsilon \in \Upsilon_r\}$ is $\alpha_1 - \omega-$open, and $\{w_\epsilon : \epsilon \in \Upsilon_r^*\}$ is $\alpha_2 - \omega-$open. Now, let the set $H_r = R - \psi\left(E - \bigcup_{\epsilon \in \Upsilon_r} v_\epsilon\right)$ is a $\beta_1 - \omega-$open containing r, and the set $H_r^* = R - \psi\left(E - \bigcup_{\epsilon \in \Upsilon^*_r} w_\epsilon\right)$ is a $\beta_2 - \omega-$open containing r, where $\psi^{-1}(H_r) \subseteq \bigcup_{\epsilon \in \Upsilon_r} v_\epsilon$, $\psi^{-1}(H_r^*) \subseteq \bigcup_{\epsilon \in \Upsilon^*_r} w_\epsilon$. Let the cover $\widetilde{H} = \{H_r : r \in R\} \cup \{H_r^* : r \in R\}$ is pair$-\omega-$open of R. Since R is pair$-\omega-$lindelöf space, then $R \subseteq \bigcup_{n=1}^{\infty} (H_{r_n}) \cup \bigcup_{m=1}^{\infty} (H_{r_m}^*)$. Therefore, $\psi^{-1}(R) \subseteq \bigcup_{n=1}^{\infty} \Psi^{-1}(H_{r_n}) \cup \bigcup_{m=1}^{\infty} \psi^{-1}(H_{r_m}^*) \subseteq \cup_{n=1}^{\infty} \tilde{u}$.
That is the space E is pair$-$lindelöf.
(2) Similar to the prove of part (1).

Corollary 3.19 Let the surjection function $\psi: (E, \alpha_1, \alpha_2) \to (R, \beta_1, \beta_2)$ is pair$-\omega-$continuous. Then the next statements are hold:
(1) If (E, α_1, α_2) is pair$-\omega-$lindelöf space, then (R, β_1, β_2) is pair$-$lindelöf,
(2) If (E, α_1, α_2) is countably pair$-\omega-$compact space, then (R, β_1, β_2) is pair$-$countably compact.

Theorem 3.20 Let the surjection function $\psi: (E, \alpha_1, \alpha_2) \to (R, \beta_1, \beta_2)$ is pair$-\omega-$continuous. Then the next statements are hold:
(1) If (R, β_1, β_2) is pair$-\omega-$closed compact space, then (E, α_1, α_2) is pair$-$compact,
(2) If (R, β_1, β_2) is pair$-\omega-$closed lindelöf space, then (E, α_1, α_2) is pair$-$lindelöf,
(3) If (R, β_1, β_2) is pair$-\omega-$closed compact space, then (E, α_1, α_2) is pair$-$countably compact.

Proof We can prove this theorem in the same way as the previous theorem (3.18).

Corollary 3.21 Let the surjection function $\psi: (E, \alpha_1, \alpha_2) \to (R, \beta_1, \beta_2)$ is pair$-\omega-$continuous. Then the next statements are hold:
(1) If $(E, \alpha_1, \alpha_2)$is pair$-\omega-$closed compact space, then (R, β_1, β_2) is pair$-$compact,
(2) If $(E, \alpha_1, \alpha_2)$is pair$-\omega-$closed lindelöf space, then (R, β_1, β_2) is pair$-$lindelöf,
(3) If (E, α_1, α_2) is pair$-\omega-$closed compact space, then (R, β_1, β_2) is pair$-$countably compact.

Definition 3.22 A space (E, α_1, α_2) is called pair$-\omega - T_1$ space, if for any two distinct points p_1 and p_2 of (E, α_1, α_2), there exists two sets u and v is pair$-\omega-$open containing p_1 and p_2, respectively, where $p_2 \notin u$ and $p_1 \notin v$.

Definition 3.23 A space (E, α_1, α_2) is called pair$-\omega - T_2$ space, if for any two distinct points p_1 and p_2 of (E, α_1, α_2), there exists two sets u and v is pair$-\omega-$open, where $p_1 \in u$ and $p_2 \in v$.

Theorem 3.24 Let the injection function $\psi: (E, \alpha_1, \alpha_2) \to (R, \beta_1, \beta_2)$ is pair$-\omega-$continuous and (R, β_1, β_2) is pair$-T_1$ space. Then the space (E, α_1, α_2) is pair$-\omega - T_1$.

Proof Assume that (R, β_1, β_2) is pair$-T_1$. For any two distinct points p_1 and p_2 of (E, α_1, α_2), there exists β_1-open set u and β_2-open set v such that $\psi(p_1) \in u$, $\psi(p_2) \notin u$, $\psi(p_1) \notin v$, $\psi(p_2) \in v$. Since ψ is pair$-\omega-$continuous and injection function, then there are subsets $\psi^{-1}(u)$ of (E, α_1, α_2) is $\alpha_1 - \omega-$open and $\psi^{-1}(v)$ of (E, α_1, α_2) is $\alpha_2 - \omega-$open, such that $p_1 \in \psi^{-1}(u)$, $p_2 \notin \psi^{-1}(u)$, $p_1 \notin \psi^{-1}(v)$, $p_2 \in \psi^{-1}(v)$. Thus, the space (E, α_1, α_2) is pair$-\omega - T_1$.

Corollary 3.25 Let the injection function $\psi: (E, \alpha_1, \alpha_2) \to (R, \beta_1, \beta_2)$ is pair$-\omega-$continuous and (R, β_1, β_2) is pair$-T_1$ space. Then the space (R, β_1, β_2) is pair$-\omega - T_1$.

Theorem 3.26 Let the injection function $\psi: (E, \alpha_1, \alpha_2) \to (R, \beta_1, \beta_2)$ is pair$-\omega-$continuous and (R, β_1, β_2) is pair$-T_2$ space. Then the space (E, α_1, α_2) is pair$-\omega - T_2$.

Proof Let (E,α_1,α_2) is pair$-T_2$ space, and the injection function $\psi:(E,\alpha_1,\alpha_2)\to(R,\beta_1,\beta_2)$ is pair$-\omega$ $-$continuous, and $p_1\neq p_2$ in (R,β_1,β_2), then $\psi^{-1}(p_1)$, $\psi^{-1}(p_2)$ are two disjoint subsets of (E,α_1,α_2). Since the space (E,α_1,α_2) is pair$-T_2$, there exists two sets α_1 $-$open u of (E,α_1,α_2), and α_2 $-$open v, such that $\psi^{-1}(p_1)\subseteq u$, $\psi^{-1}(p_2)\subseteq v$, $u\cap v=\emptyset$. Since ψ is pair$-\omega$ $-$continuous and injection function, then the sets $R-\psi(E-u)$ be β_1-ω $-$open in (R,β_1,β_2) and containing p_1, $R-\psi(E-v)$ be $\beta_2-\omega$ $-$open in (R,β_1,β_2) and containing p_2, such that $[R-\psi(E-u)\cap R-\psi(E-v)]=R-[\psi(E-u)\cup\psi(E-V)]=R-\psi(E-u\cap v)=R-\psi(p)=\phi$. Thus, the space (R,β_1,β_2) is pair$-\omega-T_2$.

Corollary 3.27 Let the injection function $\psi:(E,\alpha_1,\alpha_2)\to(R,\beta_1,\beta_2)$ is pair$-\omega$ $-$continuous and (E,α_1,α_2) is pair$-T_2$ space. Then the space (R,β_1,β_2) is pair$-\omega-T_2$.

4 Relations Pairwise ω $-$Continuous Function with Forms Continuity

Observe that the pair$-\omega$ $-$continuous functions at any given point may not always be pair$-$almost continuous functions at the same point, as demonstrated by the next two examples, which also demonstrate that pair$-$almost continuous functions at any given point do not always have to be pair$-\omega$ $-$continuous at the same point.

Example 4.1 Let the function $\psi:(\mathbb{R},\alpha_u,\alpha_u)\to(\mathbb{R},\alpha_u,\alpha_u)$ defined as $\psi(q)=0$, if q is rational number and $\psi(q)=1$ if q is irrational number. Then the function ψ is pair$-$almost continuous at any rational number. Nevertheless, the function ψ is not pair$-\omega$ $-$continuous at each real number.

Example 4.2 Let the real number set be denoted by $\mathbb{R}$, and represents the usual topology on $\mathbb{R}$ by α_u. Assume that $E=\{c,d\}$ with $\alpha=\{\emptyset,E,\{c\}\}$. Now, let $\psi:(\mathbb{R},\alpha_u,\alpha_u)\to(E,\alpha,\alpha)$ is a function such that $\psi(q)=c$ for irrational q and $\psi(q)=d$ for rational q. Then, for every rational number, ψ is a pair$-\omega$ $-$continuous function. At any real number, the function ψ cannot be pair$-$almost continuous.

Observe that the pair$-\omega$ $-$continuous functions at any given point may not always be pair$-$weakly continuous functions at the exact same point, as demonstrated by the next two examples, which also demonstrate that pair$-$weakly continuous functions at any given point do not always have to be pair$-\omega$ $-$continuous at the exact same point.

Example 4.3 At every irrational number, the pair$-\omega$ $-$continuous function ψ, as defined in example [4.1], exists. Nevertheless, the function ψ cannot be pair$-$weakly continuous.

Example 4.4 Let the real number set be denoted by $\mathbb{R}$, and represents the co-countable topology on $\mathbb{R}$ by α_{coc}. Assume that $E=\{c,d,f\}$ with $\alpha=\{\emptyset,E,\{c\},\{f\},\{c,f\}\}$. Now, let $\psi:(\mathbb{R},\alpha_u,\alpha_u)\to(E,\alpha,\alpha)$ is a function such that $\psi(q)=c$ for rational q and $\psi(q)=d$ for irrational q. Then, for any irrational number, ψ is a pair$-weakly$ continuous function. At any real number, the function ψ cannot be pair$-\omega$ $-$continuous.

Definition 4.5 A function $\psi:(E,\alpha_1,\alpha_2)\to(R,\beta_1,\beta_2)$ is called pair$-$sequentially continuous if for any pair$-$sequence $(q_{ni})\in E$ such that $(q_{ni})\to q$ we have $\psi(q_{ni})\to\psi(q)$, for $i=1,2$.

Note that the function that is defined in example [4.5] is not pair$-\omega$ $-$continuous; more so, it is pair$-$weakly and pair$-$sequentially continuous. Conversely, the next example demonstrates that a pair$-\omega$ $-$continuous function does not necessarily imply a pair$-$sequentially continuous function.

Example 4.6 Let the function $\psi:(\mathbb{R},\alpha_u,\alpha_u)\to(\mathbb{R},\alpha_{coc},\alpha_{coc})$ is identity. Then, the function ψ is pair$-$continuous (pair$-\omega$ $-$continuous). But the function ψ cannot be pair$-$sequentially continuous.
$(\bigcup_{i=1}^{n}(U(x_i)))\cap C=\phi$. Thus $\{v_\alpha:\alpha\in\Delta^*\}$ is a finite subcover of $\mathcal{V}$ for C. Hence C is α_W^i $-$compact.

5 Pairwise ω $-$Barely Continuous Functions and Pairwise Lindelöf Space

Theorem 5.1 The following are hold for every bitopological space (E,α_1,α_2):
(1) the space (E,α_1,α_2) is pair-lindelöf,
(2) any pair$-\omega$ $-$open cover of E contains a countable pair$-$subcover.

Proof (1) $\Rightarrow$ (2) Assume that $\tilde{u}=\{u_\epsilon:\epsilon\in\Upsilon\}$ is pair$-\omega$ $-$open cover of (E,α_1,α_2), since (E,α_1,α_2) is pair$-$lindelöf, so we have countable subsets Υ_q, Υ_q of the family Υ, then contains

pair−open cover $\tilde{v}=\{v_\epsilon:\epsilon\in\Upsilon_q\}\cup\{w_\epsilon:\epsilon\in\Upsilon_q^*\}$, where $\{v_\epsilon:\epsilon\in\Upsilon_q\}$ is α_1 −open, $\{w_\epsilon:\epsilon\in\Upsilon_q^*\}$ is α_2 −open, for any $\epsilon\in\Upsilon_q$, v_ϵ / u_ϵ is countable; $\epsilon\in\Upsilon_q^*$, and w_ϵ / u_ϵ is countable. Now, the space E is pair−lindelöf gives that $\tilde{v}$ contains a countable pair−subcover$\{\{v_{\epsilon_n}\}_{n=1}^\infty:\epsilon\in\Upsilon_q\}\cup\{\{w_{\epsilon_n}\}_{n=1}^\infty:\epsilon\in\Upsilon_q^*\}$. Therefore, the set $\{u_{\epsilon_n}\}_{n=1}^\infty$ covers E by $\{v_{\epsilon_n}/u_{\epsilon_n}\}_{n=1}^\infty\cup\{w_{\epsilon_n}/u_{\epsilon_n}\}_{n=1}^\infty$ but the set $\bigcup_{n=1}^\infty\{\{v_{\epsilon_n}/u_{\epsilon_n}\}\cup\{v_{\epsilon_n}/u_{\epsilon_n}\}\}$ is countable, thus E is covered with countably members of $\tilde{u}$. Therefore, we get the result.
(2) $\Rightarrow$ (1) is obviously.

Theorem 5.2 Let the function $\psi:(E,\alpha_1,\alpha_2)\to(R,\beta_1,\beta_2)$ is pair−ω −continuous. If (E,α_1,α_2) is pair−lindelöf, then (R,β_1,β_2) is so.

Proof Let $\tilde{v}=\{v_\epsilon:\epsilon\in\Upsilon_q\}\cup\{w_\epsilon:\epsilon\in\gamma_q^*\}$ be any pair−open cover of (R,β_1,β_2), where the set $\{v_\epsilon:\epsilon\in\Upsilon_q\}$ is β_1 −open and $\{w_\epsilon:\epsilon\in\Upsilon_q^*\}$ is β_2 −open. Since the function ψ is pair−ω −continuous, then $\tilde{u}=\{\psi^{-1}(v_\epsilon):\epsilon\in\Upsilon_q\}\cup\{\psi^{-1}(w_\epsilon):\epsilon\in\Upsilon_q^*\}$ is pair−ω −open cover of the space (E,α_1,α_2), which the set $\{\psi^{-1}(v_\epsilon):\epsilon\in\Upsilon_q\}$ is $\alpha_1-\omega$ −open, and $\{\psi^{-1}(w_\epsilon):\epsilon\in\Upsilon_q^*\}$ is $\alpha_2-\omega$ −open. Now, since (E,α_1,α_2) is pair−lindelöf, by theorem (5.1) we have $\tilde{u}$ contains a countable pair−subcover such that $\{\{\psi^{-1}(v_\epsilon)\}_{n=1}^\infty:\epsilon\in\Upsilon_q\}\cup\{\{\psi^{-1}(w_\epsilon)\}_{n=1}^\infty:\epsilon\in\Upsilon_q^*\}$. Therefore, $\{\{v_\epsilon\}_{n=1}^\infty:\epsilon\in\Upsilon_q\}\cup\{\{w_\epsilon\}_{n=1}^\infty:\epsilon\in\Upsilon_q^*\}$ is countably pair−subcover of $\tilde{v}$. Thus, the result.

Corollary 5.3 Let the function $\psi:(E,\alpha_1,\alpha_2)\to(R,\beta_1,\beta_2)$ is pair−ω −continuous. If (R,β_1,β_2) is pair−lindelöf, then (E,α_1,α_2) is so.

Corollary 5.4 Let the function $\psi:(E,\alpha_1,\alpha_2)\to(R,\beta_1,\beta_2)$ is pair−continuous. If (E,α_1,α_2) is pair−lindelöf, then (R,β_1,β_2) is so.

Theorem 5.5 The following are hold for every bitopological space (E,α_1,α_2):
(1) the space (E,α_1,α_2) is pair-lindelöf,
(2) any family $\{A_\epsilon:\epsilon\in\Upsilon\}$ pair−ω −closed sets of E which contains a countable intersection property that has nonempty intersection.

Definition 5.6 The function $\psi:(E,\alpha_1,\alpha_2)\to(R,\beta_1,\beta_2)$ is called pair−barely continuous if for any nonempty pair−closed $E_1\subset(E,\alpha_1,\alpha_2)$, the restriction $\psi/_{E_1}$ contains there exists one point of pair−continuity.

Definition 5.7 The function $\psi:(E,\alpha_1,\alpha_2)\to(R,\beta_1,\beta_2)$ is called pair−ω −barely continuous if for any nonempty pair−closed $E_1\subset(E,\alpha_1,\alpha_2)$, the restriction $\psi/_{E_1}$ contains there exists one point of pair−ω −continuity.

Theorem 5.8 Let the function $\psi:(E,\alpha_1,\alpha_2)\to(R,\beta_1,\beta_2)$ is pair−barely ω −continuous. If the space (E,α_1,α_2) is a hereditarily lindelöf, then (R,β_1,β_2) is pair−lindelöf.

Proof Assume that $\tilde{v}=\{v_\epsilon:\epsilon\in\Upsilon_q\}\cup\{w_\epsilon:\epsilon\in\Upsilon_q^*\}$ is pair−open cover of (R,β_1,β_2), which the sets $\{v_\epsilon:\epsilon\in\Upsilon_q\}$ is β_1 −open, $\{w_\epsilon:\epsilon\in\Upsilon_q^*\}$ is β_2 −open and $\tilde{u}=\{\psi^{-1}(v_\epsilon):\epsilon\in\Upsilon_q\}\cup\{\psi^{-1}(w_\epsilon):\epsilon\in\Upsilon_q^*\}$ is pair−open cover of the space (E,α_1,α_2), which $\{\psi^{-1}(v_\epsilon):\epsilon\in\Upsilon_q\}$ is α_1 −open, $\{\psi^{-1}(v_\epsilon):\epsilon\in{\Upsilon^*}_q\}$ is α_2 −open set. Now, let $\tilde{g}=\{g_\theta:\theta\in\Theta_1\}\cup\{g'_\theta:\theta\in\Theta_2\}$, the set $\{g_\theta:\theta\in\Theta_1\}$ is α_1 −open, $\{g'_\theta:\theta\in\Theta_2\}$ is α_2 −open, $\Theta=\Theta_1\cup\Theta_2$ where $\tilde{g}$ is pair−open subset of (E,α_1,α_2) that covered by a countably numbers of members of $\tilde{u}$. Also, let $\tilde{g}^*=\bigcup_{\theta\in\Theta_1}g_\theta\cup\bigcup_{\theta\in\Theta_2}g'_\theta$. Since the space (E,α_1,α_2) is hereditarly lindelöf, we get $\tilde{g}^*\in\tilde{g}$. Thus, the elements of g^* is maximal of $\tilde{g}$. Assume that $g^*\neq E$, then the nonempty subset of (E,α_1,α_2) that $\theta=E/_{g^*}$ is pair−closed. Thus, the function $\psi/_\theta$ is pair−ω −continuous at some $p\in\theta$. Now, let $v\in\tilde{v}$ such as $\psi(p)\in v$, then there a set N is pair−ω −open containing p as $\psi(N\cap\theta)\subset v$. Additionally, there is a set L is pair−open containing p as $L/_N$ is countable. Therefore, $L\cup\tilde{g}^*$ is covered with countably numbers of members of $\tilde{u}$. Thus, $L\cup\tilde{g}^*\epsilon\tilde{g}$ that is contradiction with g^* is a maximal element of $\tilde{g}$, $\tilde{g}^*=E$. So, E is covered by countably subcolletion of $\tilde{u}$. Hence, $\tilde{v}$ contains countable subcover, and so the space (R,β_1,β_2) is pair−lindelöf.

Corollary 5.9 Let the function $\psi:(E,\alpha_1,\alpha_2)\to(R,\beta_1,\beta_2)$ is pair-barely continuous, then if the space (E,α_1,α_2) is a hereditarily lindelöf, then (R,β_1,β_2) is pair−lindelöf.

Remark 5.10 Notice that although the function in example (4.1) is pair−barely ω −continuous, it cannot be pair−barely continuous.

6 Conclusions

As we have seen in this research, the functions of $\omega -$continuous are a generalization of continuous functions. They have a way to hold onto sequence limits and are specified on topological spaces. This implies that the image of a series under the function will likewise converge to the image of the point if it contains a sequence of points in the function's domain that converge to a point. It's a means of applying the idea of continuity to more complex circumstances such as weakened these functions, so based on that we get and explored in this research their master features, such that the concepts of pairwise $\omega -$continuous as well as pairwise barely continuous functions. We have looked at these ideas' key characteristics and revealed how they relate to other circumstances. We identified their principal characteristics as a whole and defined the conditions that must be met in order to attain comparable connections between them. We talked about their main traits and demonstrated how they work together. The report also highlighted the characteristics of these functions and offered numerous instances of them. These functions will serve as a springboard for research into all of these functions' possible futures. Future studies might look into investigating other variations of these roles such as in fuzzy, soft and group, [30], [31], [32].

Acknowledgements:

We deeply appreciate each person who contributed to this study. Their teamwork, instructions, and encouragement have been essential to finishing this study.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

- Ali Atoom: presented the main idea of the research, provided the basic definition of the research, and enriched it with several theories and illustrative examples.
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- Nabeela Abu-Alkishik: conducted scientific and linguistic scrutiny of the research, as well as reviewed and verified the academic references.

Sources of Funding for Research Presented in a Scientific Article or Scientific Article Itself

This research was funded by Ajloun National University.

Conflict of Interest

The authors acknowledge the work's authorship and declare that no personal gain is intended from its publication. The primary aim of this study is to make a scientific contribution to the area of general topology. Furthermore, every researcher certifies that this work was not published, submitted to, or accepted for publication in any journal and that it was entirely new research.