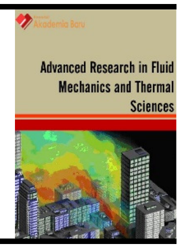
Numerical Solution of Micropolar Casson Fluid Behaviour on Steady MHD Natural Convective Flow about A Solid Sphere

Dr: Hamzeh Taha Alkasasbeh
Ajlun National Private University

Penerbit Akademia Baru

Journal of Advanced Research in Fluid Mechanics and Thermal Sciences

Journal homepage: www.akademiabaru.com/arfmts.html
ISSN: 2289-7879



Numerical Solution of Micropolar Casson Fluid Behaviour on Steady MHD Natural Convective Flow about A Solid Sphere

Open Access

Hamzeh Taha Alkasasbeh[1,*]

[1] Department of Mathematics, Faculty of Science, Ajloun National University, P.O. Box 43, Ajloun 26810, Jordan

ARTICLE INFO

Article history:
Received 15 April 2018
Received in revised form 25 July 2018
Accepted 28 July 2018
Available online 26 September 2018

ABSTRACT

In this study, steady laminar 2D (MHD) natural convection flow suspended micropolar Casson fluid over a solid sphere is investigated. The governing partial differential equations are transformed into dimensionless form and then solved numerically using an implicit finite difference scheme known as Keller-box method. The behaviors of different parameters namely, Casson fluid parameter, magnetic parameter and micropolar parameter on the local Nusselt number and the local skin friction coefficient, as well as the temperature, velocity and angular velocity have been examined graphically. From graphical results, it is found that as Casson parameter increases the local Nusselt number and angular velocity profiles increase and local skin friction coefficient, temperature and velocity profiles decrease.

Keywords:
Magnetohydrodynamic (MHD), microploar Casson fluid, numerical solution, sold sphere

1. Introduction

The micropolar fluid, a subclass of microfluids which exhibit certain microscopic effects arising from the local structure and micro-motions of the fluid elements. Eringen [1] was the first who introduce the theory of micropolar fluids describe the micro-rotation and micro-inertia effects. In the last few years, many researchers have investigated and reported these effects on heat transfer flow problems in fluid dynamics. Agarwal *et al.*, [2] studied the micropolar fluid flow with heat transfer over a porous stationary wall. Rebhi *et al.*, [3] considered the natural convection flow of a micropolar fluid over a moving vertical porous plate with chemical reaction and absorption or heat generation effects. Usman *et al.*, [4] investigated unsteady free convection flow and mass transfer in a micropolar fluid from vertical permeable surface with variable suction. The analytic solutions of resultant equations are obtained via perturbation method. The effects of viscous dissipation and joule heating on free convection flow of a micropolar fluid with constant heat and mass fluxes have been described by Haque *et al.*, [5]. They used Nachtsheim-Swigert iteration approach as the main tool to obtain numerical solutions. Srinivasacharya and Upendar [6,7] studied the micropolar fluid

* *Corresponding author.*
E-mail address: hamzahtahak@yahoo.com (Hamzeh Taha Alkasasbeh)

heat and mass transfer flow over a vertical plate with out and with chemical reaction, respectively. Sheikholeslami *et al*., [8] discussed the mass transfer in a micropolar fluid flow through a porous channel with stationary walls by Differential Transformation Method. Fakour *et al*., [9] also analyzed the heat and mass transfer flow of a micropolar fluid in a channel with permeable walls. They also compared their analytic results with numerical results obtained by Runge Kutta fourth-order method. Hussanan *et al*., [10,11] studied analytically the micropolar fluid flow subject to Newtonian heating over a vertical plate with and without taking into account the thermal radiation effects, respectively. Recently, viscoelasticity based micropolar fluid over a stretching sheet with slip condition is considered by Sui *et al*., [12]. Swalmeh *et al*., [13] study the numerical solution free convection boundary layer flow over a solid sphere in micropolar nanofluid with prescribed wall temperature using Keller-Box method.

Among the class of several other non-Newtonian fluid models, Casson fluids in the presence of heat transfer is widely used in the processing of chocolate, foams, syrups, nail, toffee and many other foodstuffs [14]. Casson [15], in his pioneering work introduced this model to simulate industrial inks. Later on, a substantial study has been done on the Casson fluid flow because of its important engineering applications. Mustafa *et al*., [16] have studied the heat transfer flow of a Casson fluid over an impulsive motion of the plate using the homotopy method. The exact solution of forced convection boundary layer Casson fluid flow toward a linearly stretching surface with transpiration effects are reported by Mukhopadhyay *et al*., [17]. In the same year, Subba *et al*., [18] considered the velocity and thermal slip conditions on the laminar boundary layer heat transfer flow of a Casson fluid past a vertical plate. Mahdy and Ahmed [19] studied the effect of magnetohydrodynamic on a mixed convection boundary flow of an incompressible Casson fluid in the stagnation point of an impulsively rotating sphere. The convective boundary layer flow of Casson nanofluid from an isothermal sphere surface is presented by Nagendra *et al*., [20]. Mehmood *et al*., [21] investigated the micropolar Casson fluid on mixed convection flow induced by a stretching sheet. Shehzad et al [22] discussed the viscous chemical reaction effects on the MHD flow of a Casson fluid over a porous stretching sheet. Recently, Khalid *et al*., [23] developed exact solutions for unsteady MHD free convection flow of a Casson fluid past an oscillating plate. Amongst the various investigations on Casson fluid, the reader is referred to some new attempts made in Qasim and Noreen [24], Hussanan *et al*., [25], Haq *et al*., [26], Manjunatha, and Choudhary [27], Aman *et al*., [28], and Alkasasbeh [29], and the references therein.

On the other hand, the flow and heat transfer phenomena about solid sphere have many important topic due to numerous engineering and industrial applications such as solving the cooling problems in turbine blades, manufacturing processes and electronic systems [30]. The exact analysis of the laminar free convection from a sphere by considering prescribed surface temperature and surface heat flux was first investigated by Chiang *et al*., [31]. Nazar *et al*., [32,33], Salleh *et al*., [34] and Alkasasbeh *et al*., [35] studied the free convection boundary layer flow on a solid sphere in a micropolar fluid with constant wall temperature, constant surface heat flux, Newtonian heating and convective boundary conditions, respectively.

Based on the above contribution, the aim of present study is to investigate the effect of MHD on free convective boundary layer flow about a solid sphere in a micropolar Casson fluid and this problem has to the author knowledge not appeared thus far in the scientific literature .

2. Mathematical Analysis

Consider a heated sphere of radius a, which is immersed in incompressible micropolar Casson fluid of ambient temperature T_∞. It is assumed that the surface temperature of the sphere is T_w

where $T_w > T_\infty$. The coordinates $\bar{x}$ and $\bar{y}$ are chosen such that $\bar{x}$ measures the distance along the surface of the sphere from the lower stagnation point and $\bar{y}$ measures the distance normal to the surface of the sphere. The constitutive relationship for an incompressible Casson fluid flow, reported by Mukhopadhyay *et al.*, [17].

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + p_y\sqrt{2\pi}\right)e_{ij}\pi > \pi_c, \\ 2\left(\mu_B + p_y\sqrt{2\pi_c}\right)e_{ij}\pi < \pi_c, \end{cases}$$

where $\pi = e_{ij}e_{ij}$, e_{ij} is the (i,j) -th component of the deformation rate, μ_B is the plastic dynamic viscosity of the non-Newtonian fluid,π_c is a critical value of this product based on the non-Newtonian model and p_y is the yield stress of the fluid. Introducing the boundary layer approximations, the continuity, momentum, microrotation and energy equations, respectively can be written as follows.

$$\frac{\partial}{\partial \bar{x}}(\bar{r}\bar{u}) + \frac{\partial}{\partial \bar{y}}(\bar{r}\bar{v}) = 0, \tag{1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \frac{(\mu+\kappa)}{\rho}\left(1+\frac{1}{\beta}\right)\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + gB(T-T_\infty)\,sin\left(\frac{\bar{x}}{a}\right) + \frac{\kappa}{\rho}\frac{\partial \bar{H}}{\partial \bar{y}} - \frac{\sigma B^2}{\rho}\bar{u}, \tag{2}$$

$$\bar{u}\frac{\partial \bar{H}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{H}}{\partial \bar{y}} = \frac{\phi}{\rho j}\frac{\partial^2 \bar{H}}{\partial \bar{y}^2} - \frac{\kappa}{\rho j}\left(2\bar{H} + \frac{\partial \bar{u}}{\partial \bar{y}}\right), \tag{3}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \alpha\frac{\partial^2 T}{\partial \bar{y}^2}. \tag{4}$$

These equations are subjected to the boundary conditions [32],

$$\bar{u} = \bar{v} = 0, T = T_w,\ \bar{H} = -\frac{1}{2}\frac{\partial \bar{u}}{\partial \bar{y}} \text{ as } \bar{y} = 0,$$

$$\bar{u} \to 0, T \to T_\infty, H \to 0, \text{ as } \bar{y} \to \infty, \tag{5}$$

where $\bar{u}$ and $\bar{v}$ are the velocity components along the $\bar{x}$and $\bar{y}$ directions, respectively, $\bar{H}$ is the angular velocity of micropolar fluid, κ is the vortex viscosity, T is the local temperature, g is the gravity acceleration, k is the thermal conductivity, σ is the electric conductivity,α is the thermal diffusivity, B is the thermal expansion coefficient, ${B_0}^2$ is magnetic field strength, ν is the kinematic viscosity, μ is the dynamic viscosity, ρ is the fluid density, $j = a^2/\sqrt{Gr}$ is the microinertia density and $\beta = \mu_B\sqrt{2\pi_c}/p_y$ is the parameter of the Casson fluid. Let $\bar{r}(\bar{x}) = a\,sin(\,\bar{x}/a)$ be the radial distance from the symmetrical axis to the surface of the sphere and we assume that the spin gradient viscosity ϕ are given by

$$\phi = (\mu\ +\ \kappa/2)j. \tag{6}$$

We introduce now the following non-dimensional variables [32],

$$x = \frac{\bar{x}}{a}, y = \frac{\sqrt[4]{Gr}}{a}\bar{y}, r = \frac{\bar{r}}{a},$$

$$u = \frac{a}{\nu\sqrt{Gr}}\bar{u},\ v = \frac{a}{\nu\sqrt[4]{Gr}}\bar{v},\ H = \frac{a^2}{\nu\sqrt[4]{Gr^3}}\bar{H},$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \tag{7}$$

where $Gr = gB(T_w - T_\infty)a^3/\nu^2$ is the Grashof number. Substituting variables (7) into equations (1)–(4), we obtain the following non-dimensional equations of the problem under consideration.

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial y}(rv) = 0, \tag{8}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = (1 + K + \tfrac{1}{\beta})\frac{\partial^2 u}{\partial y^2} + \theta\sin x - Mu + K\frac{\partial H}{\partial y}, \tag{9}$$

$$u\frac{\partial H}{\partial x} + v\frac{\partial H}{\partial y} = -K\left(2H + \frac{\partial u}{\partial y}\right) + \left(1 + \frac{K}{2}\right)\frac{\partial^2 H}{\partial y^2}, \tag{10}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2}, \tag{11}$$

where $K = \kappa/\mu$ is the material or micropolar parameter, $Pr = \nu/\alpha$ is the Prandtl number and $M = \sigma {B_0}^2 a^2/\nu\rho\sqrt{Gr}$ is the magnetic parameter. The boundary conditions (5) become

$$u = v = 0,\ \theta = 1,\ H = -\frac{1}{2}\frac{\partial u}{\partial y} \text{ at } y = 0,$$

$$u \to 0,\ \theta \to 0,\ H \to 0 \text{ as } y \to \infty. \tag{12}$$

To solve equations (8) to (11), subjected to the boundary conditions (12), we assume the following variables.

$$\psi = xr(x)f(x,y),\ \theta = \theta(x,y),\ H = xh(x,y), \tag{13}$$

where ψ is the stream function defined as

$$u = \frac{1}{r}\frac{\partial \psi}{\partial y} \text{ and } v = -\frac{1}{r}\frac{\partial \psi}{\partial x}, \tag{14}$$

which satisfies the continuity equation (10). Thus, (11) to (13) become

$$(1 + K + \tfrac{1}{\beta})\frac{\partial^3 f}{\partial y^3} + (1 + x\cot x)f\frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial f}{\partial y}\right)^2 + \frac{\sin x}{x}\theta - M\frac{\partial f}{\partial y} + K\frac{\partial h}{\partial y} = x\left(\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x \partial y} - \frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}\right), \tag{15}$$

$$\left(1 + \frac{K}{2}\right)\frac{\partial^2 h}{\partial y^2} + (1 + x\cot x)f\frac{\partial h}{\partial y} - \frac{\partial f}{\partial y}h - K\left(2h + \frac{\partial^2 f}{\partial y^2}\right) = x\left(\frac{\partial f}{\partial y}\frac{\partial h}{\partial x} - \frac{\partial f}{\partial x}\frac{\partial h}{\partial y}\right), \tag{16}$$

$$\frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} + (1 + x\cot x)f\frac{\partial \theta}{\partial y} = x\left(\frac{\partial f}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial f}{\partial x}\frac{\partial \theta}{\partial y}\right), \tag{17}$$

subject to the boundary conditions

$$f = \frac{\partial f}{\partial y} = 0, \theta = 1, h = -\frac{1}{2}\frac{\partial^2 f}{\partial y^2} \text{ at } y = 0,$$

$$\frac{\partial f}{\partial y} \to 0, \theta \to 0, h \to 0 \text{ as } y \to \infty. \tag{18}$$

It can be seen that at the lower stagnation point of the sphere, $x \approx 0$, equations (15) to (17) reduce to the following nonlinear system of ordinary differential equations.

$$(1 + K + \frac{1}{\beta})f''' + 2ff'' - f'^2 + \theta - Mf' + Kh' = 0, \tag{19}$$

$$\left(1 + \frac{K}{2}\right)h'' + 2fh' - f'h - K(2h + f'') = 0, \tag{20}$$

$$\frac{1}{Pr}\theta'' + 2f\theta' = 0. \tag{21}$$

The boundary conditions (18) become

$$f(0) = f'(0) = 0, \theta(0) = 1, h(0) = -\frac{1}{2}f''(0),$$

$$f' \to 0, \theta \to 0, h \to 0 \text{ as } y \to \infty, \tag{22}$$

where primes denote differentiation with respect to y. The physical quantities of interest in this problem are the local skin friction coefficient C_f and the Nusselt number N_u, and they can be written as

$$C_f = \frac{a^2}{\sqrt[4]{Gr^3}\mu\nu}\tau_w, N_u = \frac{a}{\sqrt[4]{Gr}k(T_w - T_\infty)}q_w, \tag{23}$$

where

$$\tau_w = \left(\mu + \frac{\kappa}{2} + \frac{P_y}{\sqrt{2\pi_c}}\right)\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)_{\bar{y}=0}, q_w = -k\left(\frac{\partial T}{\partial \bar{y}}\right)_{\bar{y}=0}. \tag{24}$$

Using the non-dimensional variables (7) and the boundary conditions (12) the local skin friction coefficient C_f and the local Nusselt number N_u are

$$C_f = \left(1 + \frac{K}{2} + \frac{1}{\beta}\right)x\left(\frac{\partial^2 f}{\partial y^2}\right)_{y=o}, \quad N_u = -\left(\frac{\partial \theta}{\partial y}\right)_{y=o}. \tag{25}$$

3. Graphical Results and Discussion

The numerical solutions of the nonlinear system of partial differential equations (15) to (17) with boundary conditions (18) are solved by the Keller-box method (KBM) with four parameters considered, namely the Prandtl number Pr, the magnetic parameter M, the micropolar parameter K and Casson parameter β. This method is an implicit finite-difference method in conjunction with Newton's method for linearization. This is a suitable method to solve parabolic partial differential equations. The boundary layer thickness $y_\infty = 14$ and step size $\Delta y = 0.02$, $\Delta x = 0.005$ are used in obtaining the numerical results. The numerical solutions start at the lower stagnation point of the sphere $x \approx 0$, with initial profiles as given by equations (19) to (21) and proceed round the sphere up to$x = 120^o$. In order to verify the accuracy of the present applied numerical scheme, a comparison with previously published results has been made. It is noticed from Table 1 that when $\mathrm{Pr} = 7$, $K = 0, 2$, $M = 0$ and $\beta \to \infty$, the results under consideration for local Nusselt number N_u reduce to the results reported by Huang and Chen [36] and Nazar *et al.*, [32] for the case of viscous and micropolar fluids respectively. It is found that the results are a good agreement. Furthermore I believe that Keller-box method is proven to be very efficient to solve this problem motion significantly.

The behavior of magnetic parameter *M* on local Nusselt number $N_u$and local skin friction C_f are seen in Figure 1 and 2. It is observed that the local Nusselt number and the local skin friction are increased with the decrease in *M*. This behavior is in accordance with the physical observation that the application of transverse magnetic field always results in a resistive type force also called Lorentz force. This type of resisting force tends to resist the fluid flow and thus reducing the fluid.

Table 1
Comparison of numerical values for the local Nusselt number N_u at Pr = 7, *K* = 0, 2, *M* = 0 and $\beta \to \infty$, for viscous value *x* with previously published results of viscous and micropolar fluid

K	0			2		
x	Huang and Chen [36]	Nazar *et al.*, [32]	Present	Huang and Chen [36]	Nazar *et al.*, [32]	Present
0°	0.9581	0.9595	0.959481	-	0.7805	0.780451
10°	0.9559	0.9572	0.957203	-	0.7787	0.778705
20°	0.9496	0.9506	0.950561	-	0.7735	0.773465
30°	0.9389	0.9397	0.939668	-	0.7650	0.765020
40°	0.9239	0.9243	0.924310	-	0.7532	0.753172
50°	0.9045	0.9045	0.904501	-	0.7378	0.737800
60°	0.8858	0.8801	0.880058	-	0.7189	0.718874
70°	0.8518	0.8510	0.851032	-	0.6964	0.696356
80°	0.8182	0.8168	0.816761	-	0.6699	0.669941
90°	0.7792	0.7792	0.779178	-	0.6393	0.639298
100°	-	-	0.732707	-	-	0.610441
110°	-	-	0.681584	-	-	0.570150
120°	-	-	0.623502	-	-	0.535707

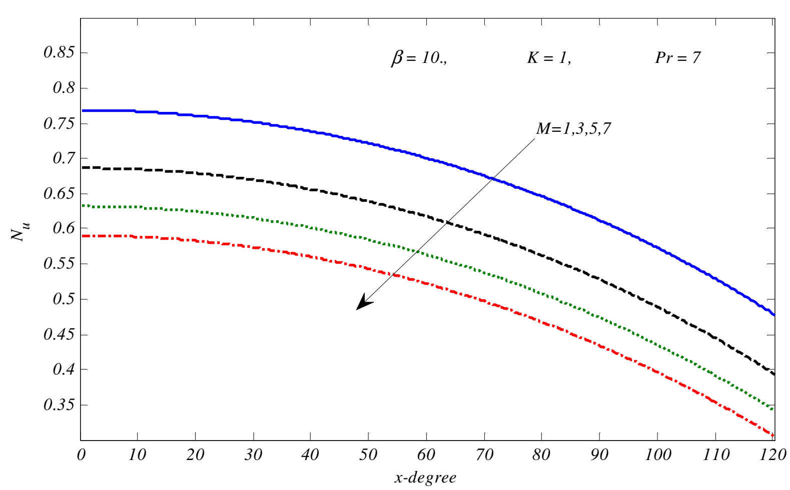


Fig. 1. Influence of M on local Nusselt number

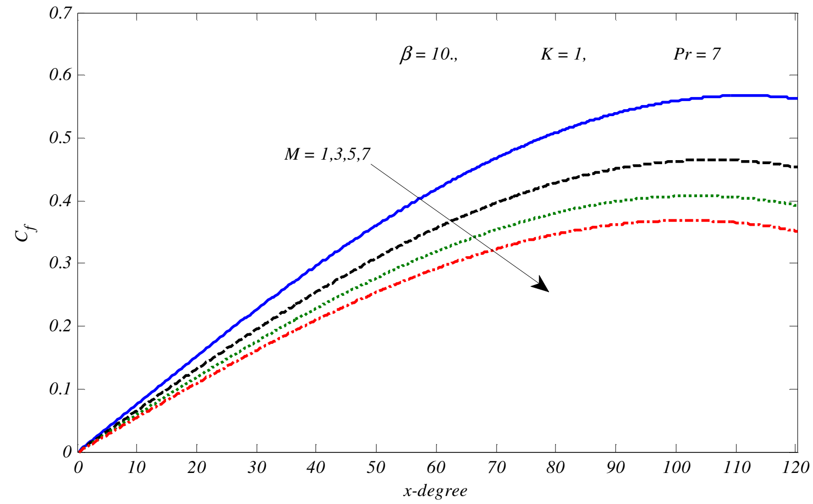


Fig. 2. Influence of M on local skin friction coefficient

Figure 3 and 4 illustrate the influence of the Casson parameter β on the local Nusselt number and the local skin friction, respectively. It is seen from these figures that an increasing of the Casson parameter leads to increases on the local Nusselt number and decreases on the local skin friction. Moreover, as the values of x increase, the rate values of the local Nusselt number decreases and the local skin friction increases.

The influence of Casson parameter on temperature, velocity and angular velocity profiles are exhibited in Figure 5-7. From figure 5 that the temperature profiles $\theta(0, y)$ increases as decreases the values of β. Figure 4 indicates that an increase in β tends to decrease in the velocity profiles $(\frac{\partial f}{\partial y})(0, y)$. It is true because is appeared in the shear term of the momentum equation (15) and an increase in implies a decrease in yield stress of the Casson fluid. Figure 7 shows that as Casson parameter β increases, the angular velocity profiles $h(0, y)$ also increases. Physically, an increase in Casson parameter means a decrease in yield stress and increases the plastic dynamic viscosity of the fluid, which makes the momentum boundary layer thicker. This effectively slows down the fluid motion.

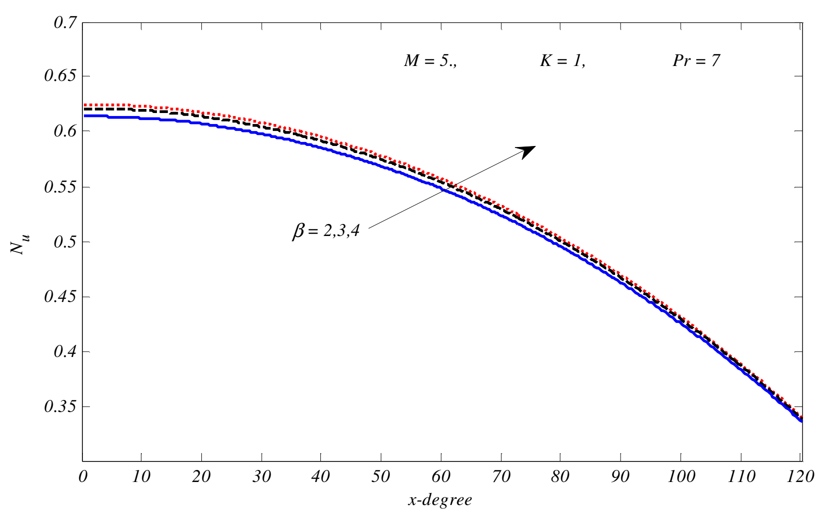


Fig. 3. Influence of β on local Nusselt number

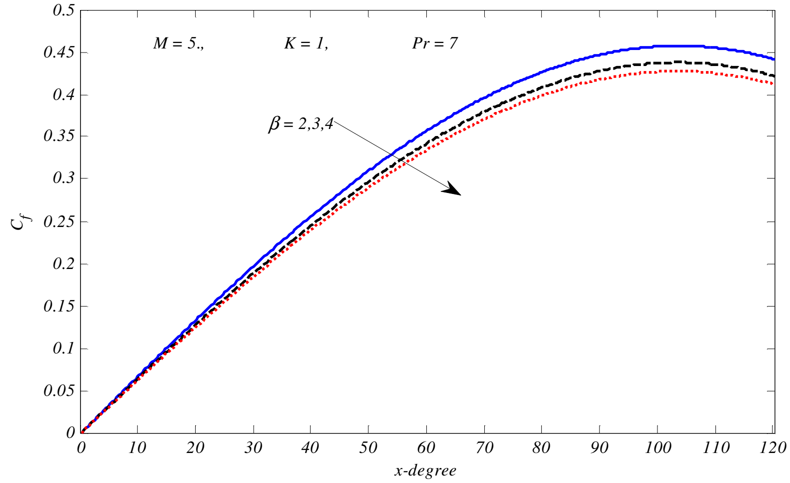


Fig. 4. Influence of β on local skin friction coefficient

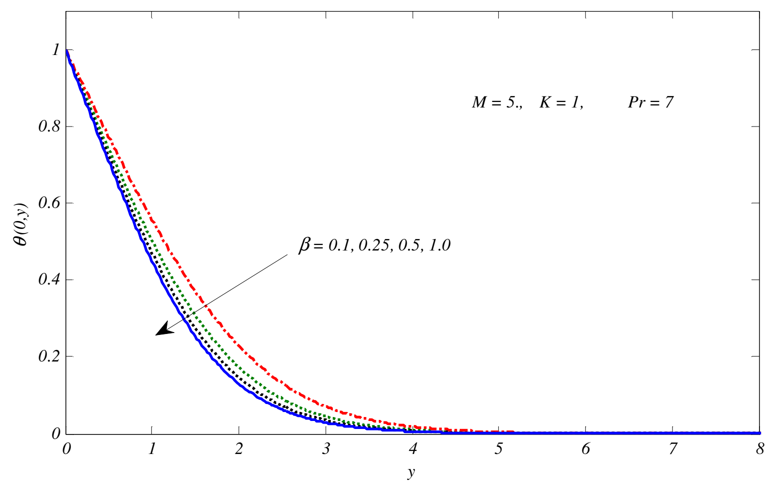


Fig. 5. Influence of β on the temperature profiles at the lower stagnation point

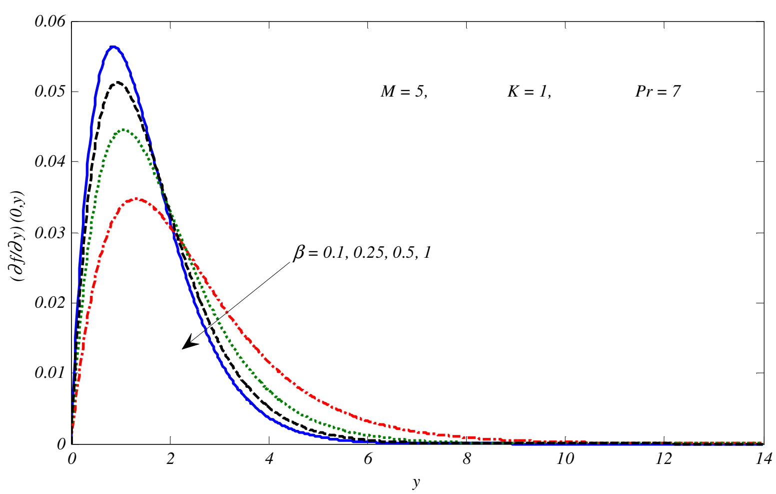


Fig. 6. Influence of β on the velocity profiles at the lower stagnation point

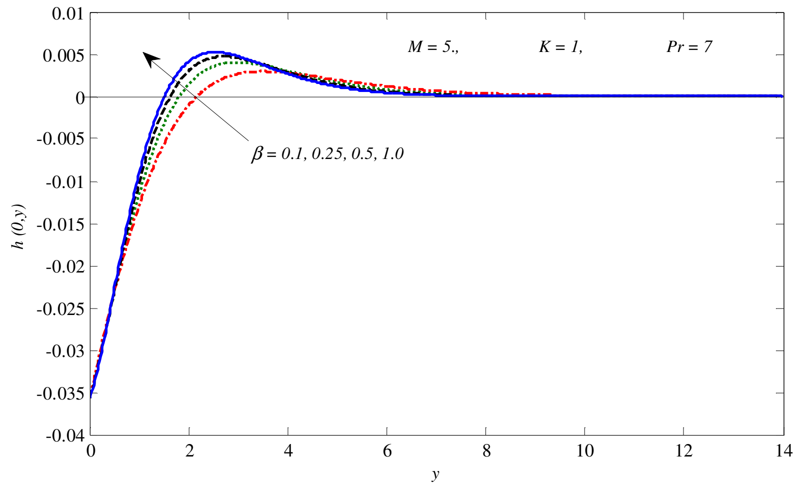


Fig. 7. Influence of β on the angular velocity profiles at the lower stagnation point

Figure 8-10 illustrate the effects of several of magnetic parameter M on temperature, velocity and angular velocity profiles. The numerical results obtained show that an increase in the magnetic parameter M the values of temperature profiles $\theta(0,y)$ increases but values of the velocity $(\frac{\partial f}{\partial y})(0,y)$ and the angular velocity profiles $h(0,y)$ decreases. This is in accordance to the physics of the problem, since the application of a transverse magnetic field results in a resistive type force (Lorentz force) similar to drag force which tends to resist the fluid flow and thus reducing its velocity and angular velocity.

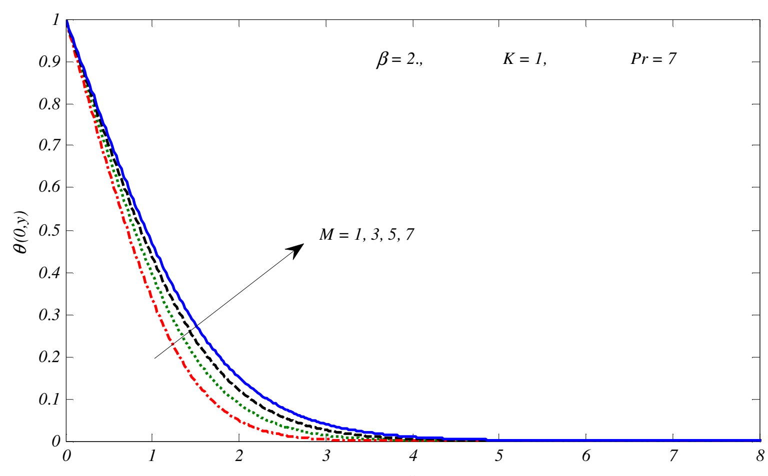


Fig. 8. Influence of M on the temperature profiles at the lower stagnation point

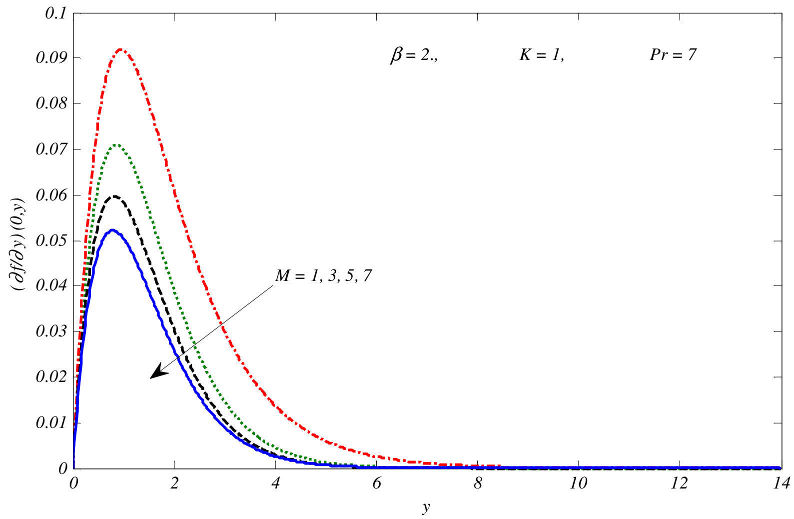


Fig. 9. Influence of M on the velocity profiles at the lower stagnation point

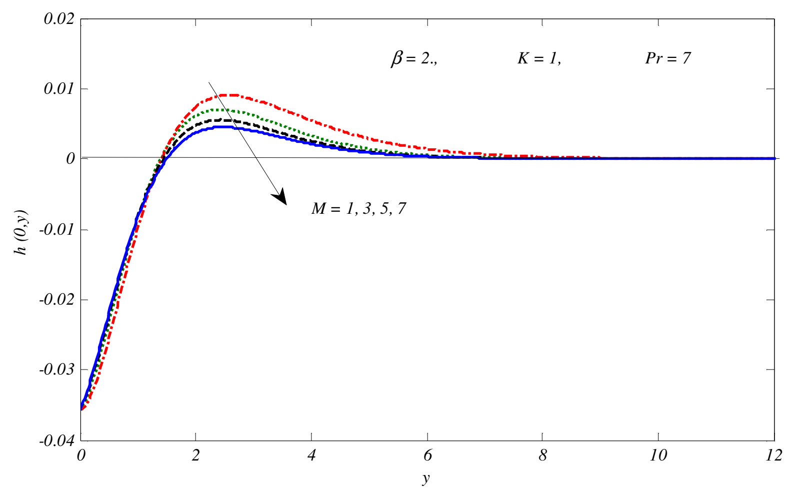


Fig. 10. Influence of M on the angular velocity profiles at the lower stagnation point

4. Conclusions

In this paper we have theoretically and numerically studied the problem of the effect of MHD free convective boundary layer flow about a solid sphere in a micropolar Casson fluid. We can conclude that, to get a physically acceptable solution.

- When the Casson parameter increases, both values of the local Nusselt number and angular velocity profiles increase, but the values of local skin friction coefficient, temperature and velocity profiles decrease.
- When the magnetic parameter increases, the values of the local Nusselt number, local skin friction coefficient, velocity and angular velocity profiles decrease, but the values of the temperature profiles increase.

Acknowledgement
The authors appreciate the constructive comments of the reviewers which led to definite improvement in the paper.

References
[1] Eringen, A. Cemal. "Theory of micropolar fluids." *Journal of Mathematics and Mechanics* (1966): 1-18.
[2] Agarwal, R. S., R. A. M. A. Bhargava, and A. V. Balaji. "Numerical solution of flow and heat transfer of a micropolar fluid at a stagnation point on a porous stationary wall." *Indian Journal of Pure and Applied Mathematics* 21 (1990): 567.
[3] Damseh, Rebhi A., M. Q. Al-Odat, Ali J. Chamkha, and Benbella A. Shannak. "Combined effect of heat generation or absorption and first-order chemical reaction on micropolar fluid flows over a uniformly stretched permeable surface." *International Journal of Thermal Sciences* 48, no. 8 (2009): 1658-1663.
[4] Usman, H., M. M. Hamza, and B. Y. Isah. "Unsteady MHD micropolar flow and mass transfer past a vertical permeable plate with variable suction." *International Journal of Com Appl*33, no. 4 (2011).
[5] Haque, Md Ziaul, Md Mahmud Alam, M. Ferdows, and A. Postelnicu. "Micropolar fluid behaviors on steady MHD free convection and mass transfer flow with constant heat and mass fluxes, joule heating and viscous dissipation." *Journal of King Saud University-Engineering Sciences* 24, no. 2 (2012): 71-84.
[6] Srinivasacharya, D., and Mendu Upendar. "Effect of double stratification on MHD free convection in a micropolar fluid." *Journal of the Egyptian Mathematical Society* 21, no. 3 (2013): 370-378.
[7] SRINIVASACHARYA, DARBHASAYANAM, and UPENDER MENDU. "Thermal radiation and chemical reaction effects on magnetohydrodynamic free convection heat and mass transfer in a micropolar fluid." *Turkish Journal of Engineering and Environmental Sciences* 38, no. 2 (2015): 184-196.
[8] Sheikholeslami, M., H. R. Ashorynejad, D. D. Ganji, and M. M. Rashidi. "Heat and mass transfer of a micropolar fluid in a porous channel." *Communications in Numerical Analysis*2014, no. unknown (2014): 1-20.
[9] Fakour, M., A. Vahabzadeh, D. D. Ganji, and M. Hatami. "Analytical study of micropolar fluid flow and heat transfer in a channel with permeable walls." *Journal of Molecular Liquids*204 (2015): 198-204.
[10] Hussanan, Abid, Mohd Zuki Salleh, Ilyas Khan, and Razman Mat Tahar. "Heat and mass transfer in a micropolar fluid with Newtonian heating: an exact analysis." *Neural Computing and Applications* 29, no. 6 (2018): 59-67.
[11] Hussanan, Abid, Mohd Zuki Salleh, Ilyas Khan, and Razman Mat Tahar. "Unsteady free convection flow of a micropolar fluid with Newtonian heating: Closed form solution." *Thermal Science* 00 (2015): 125-125.
[12] Sui, Jize, Peng Zhao, Zhengdong Cheng, and Masao Doi. "Influence of particulate thermophoresis on convection heat and mass transfer in a slip flow of a viscoelasticity-based micropolar fluid." *International Journal of Heat and Mass Transfer* 119 (2018): 40-51.
[13] Swalmeh, Mohammed Z., Hamzeh T. Alkasasbeh, Abid Hussanan, and Mustafa Mamat. "Heat transfer flow of Cu-water and Al2O3-water micropolar nanofluids about a solid sphere in the presence of natural convection using Keller-box method." *Results in Physics* 9 (2018): 717-724.
[14] Ramachandra Prasad, V., A. Subba Rao, and O. Anwar Bég. "Flow and heat transfer of casson fluid from a horizontal circular cylinder with partial slip in non-darcy porous medium." *J Appl Computat Math* 2, no. 127 (2013): 2.
[15] Casson, N. "A flow equation for pigment-oil suspensions of the printing ink type." *Rheology of disperse systems* (1959).
[16] Mustafa, M., T. Hayat, I. Pop, and A. Aziz. "Unsteady boundary layer flow of a Casson fluid due to an impulsively started moving flat plate." *Heat Transfer—Asian Research* 40, no. 6 (2011): 563-576.
[17] Mukhopadhyay, Swati, Krishnendu Bhattacharyya, and Tasawar Hayat. "Exact solutions for the flow of Casson fluid over a stretching surface with transpiration and heat transfer effects." *Chinese Physics B* 22, no. 11 (2013): 114701.
[18] Subba Rao, A., V. Ramachandra Prasad, N. Bhaskar Reddy, and O. Anwar Bég. "Heat Transfer in a Casson Rheological Fluid from a Semi-infinite Vertical Plate with Partial Slip." *Heat Transfer—Asian Research* 44, no. 3 (2015): 272-291.
[19] Mahdy, A., and Sameh E. Ahmed. "Unsteady MHD convective flow of non-Newtonian Casson fluid in the stagnation region of an impulsively rotating sphere." *Journal of Aerospace Engineering* 30, no. 5 (2017): 04017036.
[20] Nagendra, N., C. H. Amanulla, M. Narayana Reddy, A. Rao, and O. Bég. "Mathematical Study of Non-Newtonian Nanofluid Transport Phenomena from an Isothermal Sphere." *Frontiers in Heat and Mass Transfer (FHMT)* 8 (2017).
[21] Mehmood, Zaffar, R. Mehmood, and Z. Iqbal. "Numerical investigation of micropolar Casson fluid over a stretching sheet with internal heating." *Communications in Theoretical Physics* 67, no. 4 (2017): 443.
[22] Shehzad, S. A., T. Hayat, M. Qasim, and S. Asghar. "Effects of mass transfer on MHD flow of Casson fluid with chemical reaction and suction." *Brazilian Journal of Chemical Engineering* 30, no. 1 (2013): 187-195.

[23] Khalid, Asma, Ilyas Khan, Arshad Khan, and Sharidan Shafie. "Unsteady MHD free convection flow of Casson fluid past over an oscillating vertical plate embedded in a porous medium." *Engineering Science and Technology, an International Journal*18, no. 3 (2015): 309-317.

[24] Qasim, M., and S. Noreen. "Heat transfer in the boundary layer flow of a Casson fluid over a permeable shrinking sheet with viscous dissipation." *The European Physical Journal Plus*129, no. 1 (2014): 7.

[25] Hussanan, Abid, Mohd Zuki Salleh, Razman Mat Tahar, and Ilyas Khan. "Unsteady boundary layer flow and heat transfer of a Casson fluid past an oscillating vertical plate with Newtonian heating." *PloS one* 9, no. 10 (2014): e108763.

[26] Haq, Rizwan, Sohail Nadeem, Zafar Khan, and Toyin Okedayo. "Convective heat transfer and MHD effects on Casson nanofluid flow over a shrinking sheet." *Open Physics*12, no. 12 (2014): 862-871.

[27] Manjunatha, G., and Rajshekhar V. Choudhary. "Slip effects on peristaltic transport of Casson fluid in an inclined elastic tube with porous walls." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 43, no. 1 (2018): 67-80.

[28] Aman, S, Zokri, S.M , Ismail, Z, SallehM.Z, Khan, I. "Effect of MHD and Porosity on Exact Solutions and Flow of a Hybrid Casson-Nanofluid " *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 44, no. 1 (2018) 131-139

[29] Alkasasbeh, Hamzeh. "NUMERICAL SOLUTION ON HEAT TRANSFER MAGNETOHYDRODYNAMIC FLOW OF MICROPOLAR CASSON FLUID OVER A HORIZONTAL CIRCULAR CYLINDER WITH THERMAL RADIATION." *Frontiers in Heat and Mass Transfer (FHMT)* 10 (2018).

[30] Mohamed, Muhammad Khairul Anuar, Mohd Zuki Salleh, Abid Hussanan, Norhafizah Md Sarif, Nor Aida Zuraimi Md Noar, Anuar Ishak, and Basuki Widodo. "Mathematical Model of Free Convection Boundary Layer Flow on Solid Sphere with Viscous Dissipation and Thermal Radiation." *International Journal of Computing Science and Applied Mathematics* 2, no. 2 (2016): 20-25.

[31] Chiang, T., A. Ossin, and C. L. Tien. "Laminar free convection from a sphere." *Journal of Heat Transfer* 86, no. 4 (1964): 537-541.

[32] Nazar, R., N. Amin, T. Groşan, and I. Pop. "Free convection boundary layer on a sphere with constant surface heat flux in a micropolar fluid." *International communications in heat and mass Transfer* 29, no. 8 (2002): 1129-1138.

[33] Nazar, R., and N. Amin. "Free convection boundary layer on an isothermal sphere in a micropolar fluid." *International communications in heat and mass transfer* 29, no. 3 (2002): 377-386.

[34] Salleh, M. Z., R. Nazar, and I. Pop. "Numerical solutions of free convection boundary layer flow on a solid sphere with Newtonian heating in a micropolar fluid." *Meccanica* 47, no. 5 (2012): 1261-1269.

[35] Alkasasbeh, H.T., Salleh, M.Z., Tahar, R.M., Nazar, R., and Pop, I. "Effect of radiation and magnetohydrodynamic free convection boundary layer flow on a solid sphere with convective boundary conditions in a micropolar fluid." *Malaysian Journal of Mathematical Sciences*. 9, no. 3 (2015): 463-480

[36] Huang, Mingjer, and Gahokuang Chen. "Laminar free convection from a sphere with blowing and suction." *Journal of Heat Transfer (Transactions of the ASME (American Society of Mechanical Engineers), Series C);(United States)* 109, no. 2 (1987).